

$$\lim_{x \rightarrow x_0} \lim_{y \rightarrow y_0} f(x, y)$$

$$\lim_{(x, y) \rightarrow (x_0, y_0)} f(x, y)$$

$$\lim_{x \rightarrow x_0} \lim_{y \rightarrow y_0} f(x, y) \neq \lim_{y \rightarrow y_0} \lim_{x \rightarrow x_0} f(x, y)$$

9 24-18:01

$$1) \lim_{(x, y) \rightarrow (2, 2)} \frac{x+2y}{y+2x} = \frac{10}{8} = \frac{5}{4}$$

$$2) \lim_{(x, y) \rightarrow (2, 2)} \frac{x^3 - y^3}{x^2 - y^2} = \lim_{(x, y) \rightarrow (2, 2)} \frac{(x-y)(x^2+xy+y^2)}{(x-y)(x+y)} =$$

$$= \lim_{(x, y) \rightarrow (2, 2)} \frac{x^2+xy+y^2}{x+y} =$$

$$= \frac{12}{4} = 3$$

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$$5) \lim_{(x, y) \rightarrow (0, 0)} \frac{3(x^2+y^2)}{\sqrt{x^2+y^2+4} - 2} =$$

$$= \lim_{(x, y) \rightarrow (0, 0)} \frac{3(x+y) \cdot (\sqrt{x^2+y^2+4} + 2)}{x^2+y^2+4-4} =$$

$$\exists \lim_{(x, y) \rightarrow (0, 0)} (\sqrt{x^2+y^2+4} + 2) = 12$$

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$$\lim_{(x, y) \rightarrow (0, 1)} \frac{x}{x+y} = \lim_{(u, v) \rightarrow (0^+, 1)} \frac{\frac{1}{u}}{\frac{1}{u} + v} =$$

$$x \rightarrow \infty$$

$$u = \frac{1}{u}$$

$$v \rightarrow 0^+$$

$$= \lim_{(u, v) \rightarrow (0^+, 1)} \frac{1}{1+uv} =$$

$$= \lim_{(u, v) \rightarrow (0^+, 1)} \frac{1}{1+uv} = 1$$

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$$\lim_{(x, y) \rightarrow (0, 0)} \frac{x+y}{x^2+y^2} =$$

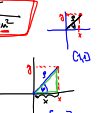
$$\lim_{(u, v) \rightarrow (0, 0)} \frac{u+v}{u^2+v^2} =$$

$$\lim_{(u, v) \rightarrow (0, 0)} \frac{u}{u^2+v^2} + \frac{v}{u^2+v^2} =$$

$$\lim_{(u, v) \rightarrow (0, 0)} \frac{u}{u^2+v^2} =$$

$$\lim_{(u, v) \rightarrow (0, 0)} \frac{u}{u^2+v^2} =$$

$$\lim_{(u, v) \rightarrow (0, 0)} \frac{u}{u^2+v^2} =$$



$$\lim_{\theta \rightarrow 0^+} \frac{e^{i\theta} \cos \theta + e^{-i\theta} \sin \theta}{e^{i\theta} \sin \theta + e^{-i\theta} \cos \theta} =$$

$$= \lim_{\theta \rightarrow 0^+} \frac{e^{i\theta} \cos \theta + e^{-i\theta} \sin \theta}{e^{i\theta} \sin \theta + e^{-i\theta} \cos \theta} =$$

$$= \lim_{\theta \rightarrow 0^+} \frac{e^{i\theta} \cos \theta + e^{-i\theta} \sin \theta}{e^{i\theta} \sin \theta + e^{-i\theta} \cos \theta} =$$

$$= \lim_{\theta \rightarrow 0^+} \frac{e^{i\theta} \cos \theta + e^{-i\theta} \sin \theta}{e^{i\theta} \sin \theta + e^{-i\theta} \cos \theta} =$$

9 24-18:18

$$\lim_{(x, y) \rightarrow (0, 0)} (x+y)^{x+y} =$$

$$x = e^{i\theta} \cos \theta$$

$$y = e^{-i\theta} \sin \theta$$

$$\lim_{\theta \rightarrow 0^+} (e^{i\theta} \cos \theta + e^{-i\theta} \sin \theta)^{e^{i\theta} \cos \theta + e^{-i\theta} \sin \theta} =$$

$$= \lim_{\theta \rightarrow 0^+} [e^{i\theta} (\cos \theta + \sin \theta)]^{e^{i\theta} \cos \theta + e^{-i\theta} \sin \theta} =$$

$$= \lim_{\theta \rightarrow 0^+} (e^{i\theta})^{e^{i\theta} \cos \theta + e^{-i\theta} \sin \theta} \cdot (\cos \theta + \sin \theta)^{e^{i\theta} \cos \theta + e^{-i\theta} \sin \theta} =$$

$$= \lim_{\theta \rightarrow 0^+} e^{i\theta (e^{i\theta} \cos \theta + e^{-i\theta} \sin \theta)} \cdot (\cos \theta + \sin \theta)^{e^{i\theta} \cos \theta + e^{-i\theta} \sin \theta} =$$

$$e^0 = 1$$

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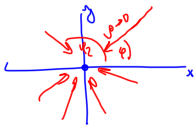
7)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x}{\sqrt{x^2+y^2}}$

$x = \rho \cos \varphi$   
 $y = \rho \sin \varphi$

$$= \lim_{\rho \rightarrow 0^+} \frac{\rho \cos \varphi}{\sqrt{\rho^2 \cos^2 \varphi + \rho^2 \sin^2 \varphi}} =$$

$$= \lim_{\rho \rightarrow 0^+} \frac{\rho \cos \varphi}{\rho} = \lim_{\rho \rightarrow 0^+} \cos \varphi =$$

Neexistuje



9 24-18:42

$\lim_{(x,y) \rightarrow (1,1)} \frac{x^2-y^4}{x^2y-1}$

$\lim_{x \rightarrow 1} \left( \lim_{y \rightarrow 1} \frac{x^2-y^4}{x^2y-1} \right) =$

$= \lim_{x \rightarrow 1} \frac{(x^2-4)}{x^2-1} = \lim_{x \rightarrow 1} \frac{(x-2)(x+2)}{(x-1)(x+1)} =$

$= \lim_{x \rightarrow 1} \frac{4}{x^2+1} = 2$

$= \lim_{y \rightarrow 1} \left( \lim_{x \rightarrow 1} \frac{x^2-y^4}{x^2y-1} \right) =$

$= \lim_{y \rightarrow 1} \frac{1-y^4}{1+y-1} = \lim_{y \rightarrow 1} \frac{y^2-4}{y-1} =$

$= \lim_{y \rightarrow 1} \frac{(y-2)(y+2)}{y-1} = \lim_{y \rightarrow 1} \frac{1}{y+1} = \frac{1}{2}$

$\lim_{x \rightarrow 1} \lim_{y \rightarrow 1} f(x,y) = 2$   
 $\lim_{y \rightarrow 1} \lim_{x \rightarrow 1} f(x,y) = \frac{1}{2}$

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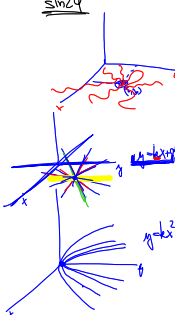
$\lim_{(x,y) \rightarrow (0,0)} \frac{2xy}{x^2+y^2}$

$x = \rho \cos \varphi$   
 $y = \rho \sin \varphi$

$$\lim_{\rho \rightarrow 0} \frac{2 \rho^2 \cos \varphi \sin \varphi}{\rho^2} =$$

$$\lim_{\rho \rightarrow 0} 2 \cos \varphi \sin \varphi = \lim_{\rho \rightarrow 0} \sin 2\varphi =$$

Neexistuje



9 24-18:54

$\lim_{(x,y) \rightarrow (0,0)} \frac{2xy}{x^2+y^2}$

$\lim_{x \rightarrow 0} \frac{2xk}{x^2+k^2} = \lim_{x \rightarrow 0} \frac{2xk}{x^2+k^2}$

$\lim_{x \rightarrow 0} \frac{2kx}{x^2+k^2} = \lim_{x \rightarrow 0} \frac{2k}{x+k} =$

$\frac{2k}{k} = 2$

$\lim_{y \rightarrow 0} \frac{2xy}{x^2+y^2} = \lim_{y \rightarrow 0} \frac{2xk}{x^2+k^2} = \lim_{y \rightarrow 0} \frac{2kx}{x^2+k^2} = \frac{2kx}{x^2+k^2}$

$\lim_{x \rightarrow 0} \frac{2kx}{x^2+k^2} = \frac{2kx}{x^2+k^2}$

$\lim_{x \rightarrow 0} \frac{2kx}{x^2+k^2} = \frac{2kx}{x^2+k^2}$

$\lim_{x \rightarrow 0} \frac{2kx}{x^2+k^2} = \frac{2kx}{x^2+k^2}$

Neexistuje

9 24-19:01

$\lim_{(x,y) \rightarrow (1,1)} \frac{2x+1}{x+y+1}$

$y = kx$

$y = k(x+\frac{1}{2}) - \frac{1}{2}$

$y = k(x-x) + \frac{1}{2}$

$y = k(x+\frac{1}{2}) - \frac{1}{2}$

$$\lim_{(x,y) \rightarrow (1,1)} \frac{2x+1}{x+y+1} =$$

$$\lim_{x \rightarrow 1} \frac{2x+1}{x+k(x+\frac{1}{2})-\frac{1}{2}+1} =$$

$$= \lim_{x \rightarrow 1} \frac{2x+1}{x+kx+\frac{k}{2}+\frac{1}{2}} =$$

$$= \lim_{x \rightarrow 1} \frac{2x+1}{(x+\frac{1}{2})(1+k)} =$$

$$= \lim_{x \rightarrow 1} \frac{2(x+\frac{1}{2})}{(x+\frac{1}{2})(1+k)} =$$

$$= 2 \lim_{x \rightarrow 1} \frac{1}{1+k} = \frac{2}{1+k}$$

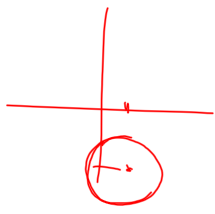
Neexistuje

9 24-19:08

$f(x,y) = \frac{5x+y}{(x-1)^2+(y+1)^2-4}$

$(x-1)^2+(y+1)^2-4 = 0$

$(x-1)^2+(y+1)^2 = 4$



9 24-19:20