① $f(x,y) = x^2 - xy + y^2$, M: $|x|+|y| \le 1$

$f'_x(x,y) = 2x - y = 0 \to y = 0$

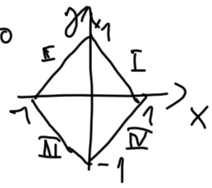
$f'_y(x,y) = -x + 2y = 0$

$x = 2y$

$x = 0$

$[0,0]$

$f(0,0) = 0$



$x + y = 1$

$x - y = 1$

$-x - y = 1$

$-x + y = 1$

I. $y = -x + 1$, $x \in \langle 0,1 \rangle$

f: $x^2 - x(-x+1) + (-x+1)^2 = x^2 + x^2 - x + x^2 - 2x + 1 = 3x^2 - 3x + 1$

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f: $3x^2 - 3x + 1 \to f': 6x - 3 = 0$

$[\frac{1}{2}, \frac{1}{2}] \to \frac{1}{4}$ $x = \frac{1}{2}$, $y = -\frac{1}{2} + 1$

II $y = x + 1$; $x \in \langle -1, 0 \rangle$

f: $x^2 - x(x+1) + (x+1)^2 = \cancel{x^2} - \cancel{x^2} - x + x^2 + 2x + 1 = x^2 + x + 1 \to f': 2x + 1 = 0$

$x = -\frac{1}{2}$, $y = \frac{1}{2}$

$f(-\frac{1}{2}, \frac{1}{2}) = (-\frac{1}{2})^2 + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$

10 29-18:11

III $y = -x - 1$, $x \in \langle -1, 0 \rangle$

f: $x^2 - x(-x-1) + (-x-1)^2 = x^2 + x^2 + x + x^2 + 2x + 1 = 3x^2 + 3x + 1 \to f': 6x + 3 = 0$

$f(-\frac{1}{2}, -\frac{1}{2}) = \frac{1}{4}$ $x = -\frac{1}{2}$

IV $y = x - 1$; $x \in \langle 0, 1 \rangle$

f: $x^2 - x(x-1) + (x-1)^2 = \cancel{x^2} - \cancel{x^2} + x + x^2 - 2x + 1 = x^2 - x + 1 \to f': 2x - 1 = 0$

$f(\frac{1}{2}, -\frac{1}{2}) = \frac{3}{4}$ $x = \frac{1}{2}$, $y = -\frac{1}{2}$

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$[1,0] \to 1$

$[0,1] \to 1$

$[-1,0] \to 1$

$[0,-1] \to 1$

ABS. MIN $f(0,0) = 0$

ABS. MAX

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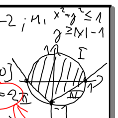
② $f(x,y) = x^2 + y^2 - xy - 2$; M: $x^2 + y^2 \le 1$, $y \ge |x| - 1$

$f'_x(x,y) = 2x - y = 0$

$f'_y(x,y) = 2y - x = 0$ $\to [0,0]$ $f(0,0) = -2$



ABS. MIN

I. $y = \sqrt{1 - x^2}$, $x \in \langle -1, 1 \rangle$

f: $\cancel{x^2} + (1 - \cancel{x^2}) - x\sqrt{1 - x^2} - 2$

f': $-[1 \cdot \sqrt{1-x^2} + x \cdot \frac{1}{2} \cdot (1-x^2)^{-\frac{1}{2}}(-2x)] = -\sqrt{1-x^2} + \frac{x^2}{\sqrt{1-x^2}} = \frac{-(1-x^2) + x^2}{\sqrt{1-x^2}} = \frac{2x^2 - 1}{\sqrt{1-x^2}} = 0$

$2x^2 = 1$

$x^2 = \frac{1}{2} \to x = \pm\frac{\sqrt{2}}{2}$

$[\pm\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}] \to -1 - (\frac{\sqrt{2}}{2})^2 = -\frac{3}{2}$

$[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}] \to -\frac{1}{2}$ ABS.MAX

II $y = -x - 1$

f: $(-x-1)^2 + x^2 - x(-x-1) - 2 = x^2 + 2x + 1 + x^2 + x^2 + x - 2 = 3x^2 + 3x - 1 \to f': 6x + 3 = 0$

$x = -\frac{1}{2}$, $y = -\frac{1}{2}$

$f(-\frac{1}{2}, -\frac{1}{2}) = -\frac{7}{4}$

III: $y = x - 1$, $x \in \langle 0, 1 \rangle$

f: $(x-1)^2 + x^2 - x(x-1) - 2 = x^2 - 2x + 1 + x^2 - x^2 + x - 2 = x^2 - x - 1 \to f': 2x - 1 = 0 \to x = \frac{1}{2}$, $y = -\frac{1}{2}$

$-\frac{5}{4}$

$[1,0] \to -1$

$[-1,0] \to -1$

$[0,-1] \to -1$

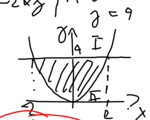
10 29-18:26

③ $f(x,y) = 2x^3 + 4x^2 + y^2 - 2xy$, M: $y = x^2$, $y = 4$

$\frac{\partial f}{\partial x} = 6x^2 + 8x - 2y = 0$

$\frac{\partial f}{\partial y} = 2y - 2x = 0$

$x = y$



$[0,0] \to 0$

$[-1,-1] \to 1$

$6x^2 + 6x = 0$

$6x(x+1) = 0$

$x_1 = 0$ $x_2 = -1$

I: $y = 4$, $x \in \langle -2, 2 \rangle$

f: $2x^3 + 4x^2 + 16 - 8x \to 6x^2 + 8x - 8 = 0$

$3x^2 + 4x - 4 = 0$

$D = 16 + 4 \cdot 3 \cdot 4 = 4 \cdot 16$

$\sqrt{D} = 8$ $x_{1,2} = \frac{-4 \pm 8}{6}$ — $\frac{2}{3}$: $[\frac{2}{3}, 4] \to$; -2: $[-2, 4] \to 32$

$2 \cdot \frac{8}{27} + 4 \cdot \frac{4}{9} + 16 - 8 \cdot \frac{2}{3}$

II. $y = x^2$, $x \in \langle -2, 2 \rangle$

f: $2\cancel{x^3} + 4x^2 + x^4 - 2\cancel{x^3} \xrightarrow{f'} 8x + 4x^3 = 0$

$4x(2 + x^2) = 0$

$[0,0] \to 0$

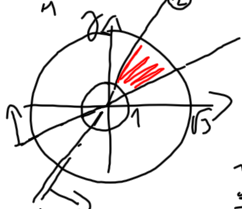
HRANICE: $[2,4] \to 32$

MAX: $[2,4]$, $[-2,4] \to 32$

MIN: $[0,0] \to 0$

10 29-18:34

(5) $\iint_M \operatorname{arctg}\left(\frac{y}{x}\right) dx\,dy$, M: $x^2+y^2 \geq 1$, ≤ 3 (2); $\frac{x}{\sqrt{3}} \leq y \leq \sqrt{3}\,x$ (1)



$X: \begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$ $\quad J = r$

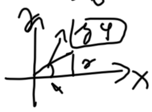
$r \in \langle 1, \sqrt{3} \rangle$

$\varphi \in \langle \frac{\pi}{6}, \frac{\pi}{3} \rangle$

$= \int_{\pi/6}^{\pi/3} \int_1^{\sqrt{3}} \operatorname{arctg} \operatorname{tg}\varphi \; r\,dr\,d\varphi = \left[\frac{\varphi^2}{2}\right]_{\pi/6}^{\pi/3} \cdot \left[\frac{r^2}{2}\right]_1^{\sqrt{3}} =$

$= \frac{1}{2}\left[\frac{\pi^2}{9} - \frac{\pi^2}{36}\right] \cdot \underbrace{\left[\frac{3}{2} - \frac{1}{2}\right]}_{1} = \frac{1}{2} \cdot \frac{3\pi^2}{36} = \frac{\pi^2}{24}$

$\frac{y}{x} = \frac{r\sin\varphi}{r\cos\varphi} = \operatorname{tg}\varphi$



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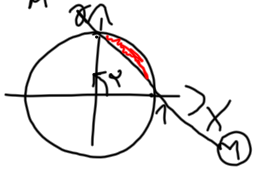
(6) $\iint_M xy\,dx\,dy$, M: $x^2+y^2 \leq 1$ $\quad r^2 \leq 1$

$x + y \geq 1$

$y \geq 1 - x$ (1)



$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$

$\varphi \in \langle 0, \frac{\pi}{2} \rangle$

(1) $r(\cos\varphi + \sin\varphi) \geq 1$

$r \geq \frac{1}{\cos\varphi + \sin\varphi}$ $\quad r \in \langle \frac{1}{\cos\varphi+\sin\varphi}, 1 \rangle$

$\int_0^{\pi/2} \int_{\frac{1}{\cos\varphi+\sin\varphi}}^{1} r\cos\varphi \, r\sin\varphi \; r\,dr\,d\varphi$

10 29-19:21

(7) $\iint_M \frac{x^2}{y^2} dx\,dy$, M: $x = 2$, $y = x$, $xy = 1 \Rightarrow y = \frac{1}{x}$

$\int_1^2 \int_{1/x}^{x} \frac{x^2}{y^2}\,dy\,dx = \int_1^2 \left[\frac{x^2 \cdot (-1)}{y}\right]_{1/x}^{x} dx =$

$\int_1^2 -\left[\frac{x^2}{x} - \frac{x^2}{\frac{1}{x}}\right] dx = \int_1^2 (x - x^3)\,dx =$

$= -\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_1^2 = -\left[\underbrace{\frac{4}{2} - \frac{16}{4}}_{-2} - \left(\underbrace{\frac{1}{2} - \frac{1}{4}}_{\frac{1}{4}}\right)\right] =$

$= \frac{9}{4}$ //

10 29-19:30