$1+x+x^2+\dots = \frac{1}{1-x}$ $\quad |x|<1$

$\{a_n\}_{n=0}^{\infty}$

$\sum_{n=0}^{\infty} a_n x^n$

$a_0+a_1x+a_2x^2+\dots$

$1+x^2+x^4+\dots = \frac{1}{1-x^2}$

1, 0, 1, 0, 1 ...

0, 1, 0, 1, 0, 1

$x+x^3+\dots = \frac{x}{1-x^2}$

0, 0, 1, 0, 1, 0, 1, ...

$x^2+x^4+\dots = \frac{x^2}{1-x^2}$

$(1-x)^r = \sum_{k\ge 0}\binom{r}{k}x^k$!

$r\in\mathbb{R}$

$\binom{r}{k} = \frac{r(r-1)(r-2)\dots(r-k+1)}{k!}$

$\frac{1}{(1-x)^n} = \binom{n-1}{n-1}+\binom{n}{n-1}x+\binom{n+1}{n-1}x^2+\dots$

12 10-17:57

P r klad 1. Kolika zp usoby je mo zn e koupit 12 bal ck u k avy, maj -li v prodejn e k avu p eti druh u?
1. od ka zd e k avy je t reba koupit alespo n dva bal cky
2. od ka zd e k avy m a b yt koupen sud y po cet bal ck u
3. jedn e z k av jsou k dispozici pouze 3 bal cky

$\underbrace{(1+x^1+x^2+x^3+x^4\dots)}_{\text{I. dr}}\cdot\underbrace{(1+x+x^2+\dots)}_{\text{II. dr}}\dots$

$= ?\,x^{12}$

$x^1\cdot x^5\cdot x^4\cdot x^2\cdot x^0$ $\qquad x^3\cdot x^2\cdot x^1\cdot x^0\cdot x^6$

1. 5 4

$x^0\cdot x^0\cdot x^0\cdot x^0\cdot x^{12}$

$= (1+x+x^2+\dots)^5 = \frac{1}{(1-x)^5}$

$\frac{1}{(1-x)^5} = \binom{4}{4}+\binom{5}{4}x+\binom{6}{4}x^2+\binom{7}{4}x^3+\dots + \binom{16}{4}x^{12}$

12 10-18:12

P r klad 1. Kolika zp usoby je mo zn e koupit 12 bal ck u k avy, maj -li v prodejn e k avu p eti druh u?
1. od ka zd e k avy je t reba koupit alespo n dva bal cky
2. od ka zd e k avy m a b yt koupen sud y po cet bal ck u
3. jedn e z k av jsou k dispozici pouze 3 bal cky

$(x^2+x^3+\dots)(x^2+x^3+\dots)\dots$ $\quad x^{12}$

$= \left(\frac{x^2}{1-x}\right)^5 = \frac{x^{10}}{(1-x)^5} = x^{10}\cdot\frac{1}{(1-x)^5}$

$\binom{6}{4}$ $\quad ?\,x^2$

$(1+x^2+x^4+\dots)(1+x^2+x^4\dots)\dots$

$= (1+x^2+x^4+\dots)^5$

$= \frac{1}{(1-x^2)^5} = \binom{4}{4}+\binom{5}{4}x^2+\binom{6}{4}x^4+\binom{7}{4}x^6+\binom{8}{4}x^8+\binom{9}{4}x^{10}+\binom{10}{4}x^{12}+\dots$

$\binom{10}{4}$

$(1+x+x^2+x^3)\cdot(1+x+x^2\dots)(1+x+x^2\dots) = (1+x+x^2+x^3)\cdot(1+x+x^2+\dots)^4 =$

$\frac{1-x^4}{1-x}\cdot\frac{1}{(1-x)^4} = \boxed{\frac{1-x^4}{(1-x)^5}}$

$\to (1-x^4)\,\boxed{\frac{1}{(1-x)^5}} \leftarrow$

$\frac{1}{(1-x)^5} = \binom{4}{4}+\binom{5}{4}x+\binom{6}{4}x^2+\dots+\binom{12}{4}x^8+\dots+\binom{16}{4}x^{12}+\dots$

$= \dots + \left[\binom{16}{4}-\binom{12}{4}\right]x^{12}\dots$

12 10-18:18

$(1+x+x^2+x^3)\cdot(1+x+x^2+\dots)(1+x+x^2+\dots)$

$= (1+x+x^2+x^3)\cdot(1+x+x^2+\dots)^4 =$

$(1+x+x^2+x^3)\cdot\frac{1}{(1-x)^4}$

$\frac{1}{(1-x)^4} = \binom{3}{3}+\binom{4}{3}x+\binom{5}{3}x^2+\dots + \binom{12}{3}x^9+\binom{13}{3}x^{10}+\binom{14}{3}x^{11}+\binom{15}{3}x^{12}+\dots$

$(1+x+x^2+x^3)\cdot\left(\binom{3}{3}+\binom{4}{3}x+\dots+\binom{12}{3}x^9+\binom{13}{3}x^{10}+\binom{14}{3}x^{11}+\binom{15}{3}x^{12}+\dots\right)$

$\binom{15}{3}+\binom{14}{3}+\binom{13}{3}+\binom{12}{3}$

12 10-18:23

P r klad 2. Ur cete kolika zp usoby je mo zn e naplnit ta sku 20 kusy uveden ych druh u ovoce, p ri cem z jednotliv e kusy t eho z druhu nerozli sujeme, nemus b yt vyu zity v sechny druhy a nav c:
1. jablek m u ze b yt libovoln y po cet
2. hru sek mus b yt sud y po cet
3. manga mohou b yt nejv y se 2
4. pomelo bude alespo n jedno

5) žádný banán

$(1+x+x^2+\dots)(1+x^2+x^4+\dots)(1+x+x^2)(x+x^2+x^3\dots) =$

x^{20}

$\frac{1}{1-x},\ \frac{1}{1-x^2}\cdot\frac{1-x^3}{1-x}\cdot\frac{x}{1-x}$

$\frac{x(1-x^3)}{(1-x)^3}\cdot\frac{1}{1-x^2}$

$* \ \frac{x-x^4}{(1-x)^3}\cdot\frac{1}{1-x^2}$ $\heartsuit$

$* = \frac{x(1-x)(1+x+x^2)}{(1-x)^3} = \frac{x(1+x+x^2)}{(1-x)^2}$

$\heartsuit = \frac{1}{1-x^2} = 1+x^2+x^4+\dots$

$\frac{1}{(1-x)^2} = \binom{1}{1}+\binom{2}{1}x+\binom{3}{1}x^2+\dots$

$x(1+x+x^2)\cdot\left[\binom{1}{1}+\binom{2}{1}x+\binom{3}{1}x^2\dots\right]\left[1+x^2+x^4\dots\right]$

x^{20}

12 10-18:34

$\frac{1}{1-x},\ \frac{1}{1-x^2}\cdot\frac{1-x^3}{1-x}\cdot\frac{x}{1-x} =$

$\frac{\cancel{x}}{(1-x)^{\cancel{3}\,4}}\cdot\frac{1-x^3}{1+x}$.

12 10-18:44

M ame v pene zence 4 korunov e mince, 5 dvoukorunov ych a 3 p etikorunov e. Z automatu, kter y nevrac , chceme Colu za 22 K c. Kolika zp usoby to um me, ani z bychom ztratili p replatek?

$$(1+x+x^2+x^3+x^4)(1+x^2+x^4+x^6+x^8+x^{10})\cdot(1+x^5+x^{10}+x^{15})$$

$$x^{22}$$

12 10-18:45

$$\begin{cases} a_0=0 \\ a_1=1 \\ a_{n+2}=a_{n+1}+a_n \end{cases}$$

$$a_n = F(n)$$

$$x^2 = x^1 + 1 \quad (2,\ 0,\ 0)$$

$$a_{n+3} = 2a_n - 3a_{n+1} + 5a_{n+2}$$

$$x^3 = 2 - 3x + 5x^2$$

i) kořen Ch. p. jsou navz. r. R $\lambda_1, \dots, \lambda_n$

ii)

$$a_n = a_1\lambda_1^n + \dots + a_n\lambda_n^n$$

$$a_n = a_1\lambda^n + a_2 n\lambda^n + \dots a_n n^n \lambda^n$$

12 10-18:47

$$x_n = 5x_{n-1} + 6x_{n-2}$$

$x_1 = 2$, $x_2 = 4$

i) Ch. p.

$$x^2 = 5x + 6$$
$$x^2 - 5x - 6 = 0$$
$$(x-6)(x+1) = 0$$
$$x_1 = 6$$
$$x_2 = -1$$

$$a_n = a\cdot 6^n + b(-1)^n$$

$n=1$, $a_1=2$, $n=2$, $a_2=4$

$$a_1 = 6a - b$$
$$2 = 6a - b \quad /\cdot 6$$
$$4 = 36a + b$$
$$6 = 42a$$
$$a = \frac{1}{7}$$
$$-8 = 7b$$
$$b = -\frac{8}{7}$$

$$a_n = \frac{1}{7}6^n - \frac{8}{7}\cdot(-1)^n$$

12 10-18:52

12 10-18:58

$$x_n = 4x_{n-1} - 4x_{n-2}$$

$x_1 = 1$, $x_2 = 4$

i) Ch. p.

$$x^2 = 4x - 4$$
$$x^2 - 4x + 4 = 0$$
$$(x-2)^2 = 0$$
$$x = 2$$

$$a_n = a\cdot 2^n + b\cdot n\cdot 2^n$$

$n=1$: $a_1 = 2a + 2b$

$n=2$: $a_2 = 4a + 8b$

$$1 = 2a + 2b$$
$$4 = 4a + 8b$$
$$2 = 4b \qquad 0 = -4a$$
$$b = \frac{1}{2} \qquad a = 0$$

$$\boxed{a_n = \frac{1}{2} n 2^n}$$

$$a_1 = \frac{1}{2}\cdot 1\cdot 2 = 1$$
$$a_2 = \frac{1}{2}\cdot 2\cdot 4 = 4$$

12 10-18:58

$$B_{n+3} = 4B_n - 8b_{n+1} + 5B_{n+2}$$

$B_0 = 0$, $B_1 = 1$, $B_2 = 2$

i) Ch. p.

$$x^3 = 4 - 8x + 5x^2$$
$$x^3 - 5x^2 + 8x - 4 = 0$$

	1	-5	8	-4
1	1	-4	4	0

$$(x-1)(x^2-4x+4) = 0$$
$$(x-1)(x-2)^2 = 0$$
$$x_1 = 1$$
$$x_{2,3} = 2$$

$n=0$, $n=1$, $n=2$

$$B_n = a\cdot(1)^n + b\cdot 2^n + c\cdot n\cdot 2^n$$

$$a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$$
$$x^3 = -3x^2 - 3x - 3$$
$$x^3 + 3x^2 + 3x + 1 = 0$$
$$(x+1)^3 = 0$$
$$a_n = a(-1)^n + b\cdot n\cdot(-1)^n + c\cdot n^2(-1)^n$$

$$B_n = a\cdot(1)^n + b\cdot 2^n + c\cdot n\cdot 2^n$$

$n=0$: $0 = a + b \Rightarrow a = -b$

$n=1$: $1 = a + 2b + 2c$

$n=2$: $2 = a + 4b + 8c$

$$1 = b + 2c$$
$$2 = 3b + 8c$$
$$-1 = 2c \Rightarrow c = -\frac{1}{2}$$
$$-2 = -b \Rightarrow b = 2$$
$$a = -2$$

$$B_n = (-2) + 2\cdot 2^n + \left(-\frac{1}{2}\right)\cdot n\cdot 2^n$$

12 10-19:03

$$\boxed{a_n = a_{n-1} + 2a_{n-2}}$$

1

n dní

a_n 1. den: pamlsek → a_{n-1}; vodítko → a_{n-2}; mrkev → a_{n-2}

$$a_n = a_{n-1} + 2a_{n-2}$$

$a_1 = 1$

$a_2 = 3$ (2p, h, v)

i) Ch.r.

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x_1 = 2$

$x_2 = -1$

$a_n = a \cdot 2^n + b(-1)^n$

n=1: $1 = 2a - b$

n=2: $3 = 4a + b$

$6a = 4 \qquad 1 = 3b$

$a = \frac{2}{3} \qquad b = \frac{1}{3}$

$$a_n = \frac{2}{3} \cdot 2^n + \frac{1}{3}(-1)^n$$

12 10-19:12

$$\boxed{A_n = \left(\frac{3-\sqrt{13}}{2}\right)^n + \left(\frac{3+\sqrt{13}}{2}\right)^n} \;!$$

$a_1 = \frac{3-\sqrt{13}}{2} + \frac{3+\sqrt{13}}{2} = 3$

$a_2 = \frac{9-6\sqrt{13}+13}{4} + \frac{9+6\sqrt{13}+13}{4}$

$= \frac{44}{4} = 11$

$$\boxed{\frac{3-\sqrt{13}}{2},\ \frac{3+\sqrt{13}}{2}}$$

$$\boxed{x^2 - 3x - 1 = 0}$$

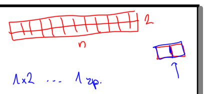
$x^2 = 3x + 1$

$a_n = 3a_{n-1} + a_{n-2}$

$a_1 = 3$

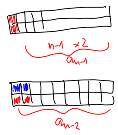
$a_2 = 11$

12 10-19:18



1×2 … 1 zp.

2×2 … 2 zb



$n-1 \times 2$, a_{n-1}

$a_n =$

a_{n-2}

$$\boxed{a_n = a_{n-1} + a_{n-2}}$$

$a_1 = 1$

$a_2 = 2$

Ch.p.:

$x^2 = x + 1$

$x^2 - x - 1 = 0$

$x_{1,2} = \frac{1 \pm \sqrt{5}}{2}$

$a_n = a \cdot \left(\frac{1+\sqrt{5}}{2}\right)^n + b\left(\frac{1-\sqrt{5}}{2}\right)^n$

n=1: $1 = a\left(\frac{1+\sqrt{5}}{2}\right) + b\left(\frac{1-\sqrt{5}}{2}\right)$

n=2: $2 = a\,\frac{6+2\sqrt{5}}{4} + b\,\frac{6-2\sqrt{5}}{4}$

$1 = a\left(\frac{1+\sqrt{5}}{2}\right) + b\left(\frac{1-\sqrt{5}}{2}\right)$

$2 = a\left(\frac{3+\sqrt{5}}{2}\right) + b\left(\frac{3-\sqrt{5}}{2}\right)$

12 10-19:23

$1 = a\left(\frac{1+\sqrt{5}}{2}\right) + b\left(\frac{1-\sqrt{5}}{2}\right)$

$2 = a\left(\frac{3+\sqrt{5}}{2}\right) + b\left(\frac{3-\sqrt{5}}{2}\right)$

$2 = a(1+\sqrt{5}) + b(1-\sqrt{5})$

$4 = a(3+\sqrt{5}) + b(3-\sqrt{5})$

$2 = a(1+\sqrt{5}) + b(1-\sqrt{5})$

$2 = 2a + 2b$

$1 = a + b$

$a = 1 - b$

$2 = (1-b)(1+\sqrt{5}) + b(1-\sqrt{5})$

$2 = 1 + \sqrt{5} - b - b\sqrt{5} + b - b\sqrt{5}$

$1 = \sqrt{5} - 2b\sqrt{5}$

$1 - \sqrt{5} = -2b$

$b = \frac{1-\sqrt{5}}{-2}$

$a = 1 - \frac{1-\sqrt{5}}{-2}$

12 10-19:29