$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}\cdot b^1 + \dots + \binom{n}{k}a^{n-k}\cdot b^k + \dots + \binom{n}{n}a^0 b^n$$

$$(a+b)^n = \underbrace{(a+b)(a+b)\dots(a+b)}_{n \text{ krát}} = \sum_{k=0}^{n} \binom{n}{k} a^{n-k}\cdot b^k$$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} = \frac{n(n-1)\cdots(n-k+1)}{k!}$$

12 12-12:11

$$(1+x)^n = \sum_{k=0}^{n}\binom{n}{k}x^k$$

zderivujeme:

$$n(1+x)^{n-1} = \sum_{k=1}^{n}\binom{n}{k}k\cdot x^{k-1}$$

$x=1$: $n\cdot 2^{n-1} = \sum_{k=1}^{n}\binom{n}{k}\cdot k = \sum_{k=0}^{n}\binom{n}{k}k$

12 12-12:20

$$(1,1,1,\dots) \longleftrightarrow \sum_{n=0}^{\infty} 1\cdot x^n = 1+x+x^2+x^3+\dots = \frac{1}{1-x}$$

lze dosazovat?

$x=2$: $1+2+4+8+\dots \overset{?}{=} \frac{1}{1-2} = -1$

$1,1,1,1,\dots \longleftrightarrow \frac{1}{1-x}$

$1,2,4,8,\dots \longleftrightarrow \sum_{n=0}^{\infty} 2^n x^n = \sum (2x)^n = \frac{1}{1-2x}$

$2^0, 2^1, 2^2$

$2,3,5,9,\dots \longleftrightarrow \frac{1}{1-x}+\frac{1}{1-2x}$

$(a_n)_{n=0}^{\infty} \longleftrightarrow \sum a_n\cdot x^n = a(x)$

$+$

$(b_n)_{n=0}^{\infty} \longleftrightarrow \sum b_n\cdot x^n$

$\parallel$

$(a_n+b_n)_{n=0}^{\infty} \longleftrightarrow \sum (a_n+b_n)\cdot x^n$

$(2\cdot a_n)_{n=0}^{\infty} \longleftrightarrow \sum 2a_n x^n = 2\cdot \sum a_n x^n = 2\cdot a(x)$

12 12-12:29

$1,1,1,\dots \longleftrightarrow \frac{1}{1-x}$

$\underbrace{0,0,0}_{3},1,1,1,1,\dots \longleftrightarrow \frac{x^3}{1-x}$

$1,1,1,1,1,\dots \longleftrightarrow \frac{1}{1-x}$

posuno 3 $\leftarrow$

$$\longleftrightarrow \frac{\frac{1}{1-x}-(1+x+x^2)}{x^3} = \frac{1+x+x^2+\dots-(1+x+x^2)}{x^3} = \frac{x^3+x^4+x^5+\dots}{x^3} = 1+x+x^2+\dots = \frac{1}{1-x}$$

12 12-12:44

$1,1,1,1,\dots \longleftrightarrow \frac{1}{1-x}$

subst. $x \leftarrow x^2$

$$\sum_{n=0}^{\infty}(x^2)^n = \frac{1}{1-x^2}$$

$\parallel$

$1+0\cdot x+1\cdot x^2+0\cdot x^3+\dots$

$\updownarrow$

$1,0,1,0,1,0,\dots$

12 12-12:47

$$(a_0,a_1,\dots) \longleftrightarrow a(x) = \sum_{n=0}^{\infty} a_n\cdot x^n$$

$$a'(x) = \sum_{n=1}^{\infty} a_n\cdot n\cdot x^{n-1} =$$

$n=k+1$

$$(a_1,2a_2,3a_3,\dots) \longleftrightarrow = \sum_{k=0}^{\infty} a_{k+1}\cdot(k+1)\cdot x^k$$

12 12-12:49

Vytvořující funkce a řešení rekurencí | Exponenciální vytvořující funkce | Pravděpodobnostní vytvořující funkce

Přehled mocninných řad:

$$\frac{1}{1-x} = \sum_{n\geq 0} x^n, \quad \text{„geom. řada"}$$

$$-\ln(1-x) = \ln\frac{1}{1-x} = \sum_{n\geq 1}\frac{x^n}{n}, \quad \ln(1-x) = -\sum_{n=1}^{\infty}\frac{x^n}{n}$$

$$e^x = \sum_{n\geq 0}\frac{x^n}{n!},$$

$$\sin x = \sum_{n\geq 0}(-1)^n\frac{x^{2n+1}}{(2n+1)!},$$

$$\cos x = \sum_{n\geq 0}(-1)^n\frac{x^{2n}}{(2n)!},$$

$$(1+x)^r = \sum_{k\geq 0}\binom{r}{k}x^k. \quad \text{zobecněná binom. věta}$$

12 12-12:53

$$\binom{\pi}{3} = \frac{\pi(\pi-1)(\pi-2)}{3!}$$

$$\binom{-2}{3} = \frac{(-2)(-3)(-4)}{3!}$$

$$(1-x)^{-n} = \sum_{k=0}^{\infty}\binom{-n}{k}\cdot(-x)^k$$

koef. u x^k: $\binom{-n}{k}\cdot(-1)^k =$

$$= \frac{(-n)(-n-1)(-n-2)\cdots(-n-k+1)}{k!}\cdot(-1)^k$$

$$= \frac{n(n+1)(n+2)\cdots(n+k-1)}{k!} = \binom{n+k-1}{k} = \binom{n+k-1}{n-1}$$

12 12-12:57

$$\begin{array}{ccccccccc} &&&& 1 &&&& \\ &&& 1 && 1 &&& \\ && 1 && 2 && 1 && \\ & 1 && 3 && 3 && 1 & \\ 1 && 4 && 6 && 4 && 1 \end{array}$$

$\binom{n}{0}$, $\binom{n}{1}$, $\binom{n}{2}$, $\binom{n}{3}$

$\frac{1}{1-x}$, $\frac{1}{(1-x)^2}$, $\frac{1}{(1-x)^3}$

12 12-13:01

$$S = a + a\cdot q + a\cdot q^2 + \dots + a\cdot q^n \quad |\cdot q$$

$$q\cdot S = a\cdot q + a\cdot q^2 + \dots + a\cdot q^n + a\cdot q^{n+1}$$

$$S - q\cdot S = a - a\cdot q^{n+1}$$

$$S(1-q) = a(1-q^{n+1})$$

$$S = a\cdot\frac{q^{n+1}-1}{q-1}$$

12 12-13:05

$F_0 = 0,\ F_1 = 1,\ F_n = F_{n-1} + F_{n-2} \quad n \geq 2$

$$\boxed{F_n = F_{n-1} + F_{n-2} + [n=1]}$$

$$\sum_{n=0}^{\infty} F_n\cdot x^n = \sum F_{n-1}x^n + \sum F_{n-2}x^n + \sum [n=1]x^n$$

$$\sum F_{n-1}x^n = x\sum_{n=0}^{\infty} F_{n-1}x^{n-1}$$

$$F(x) = x\cdot F(x) + x^2\cdot F(x) + x$$

$$F(x) = \frac{x}{1-x-x^2}$$

12 12-13:13

$$\frac{x}{1-x-x^2} = \frac{A}{x-x_1} + \frac{B}{x-x_2}$$

$$\frac{x}{1-x-x^2} = \frac{Ax - Ax_2 + Bx - Bx_1}{x^2+x-1}$$

$$\Rightarrow \quad x = -x(A+B)$$

$$1 = Ax_2 + Bx_1$$

$$F_n = a\cdot\lambda_1^n + b\cdot\lambda_2^n$$

12 12-13:20

$$x^2 + x - 1 = 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$$\lambda_1 = \frac{1}{\frac{-1+\sqrt{5}}{2}} = \frac{2}{-1+\sqrt{5}} = \frac{2(\sqrt{5}+1)}{4} = \frac{\sqrt{5}+1}{2}$$

$$\lambda_2 = \dots$$

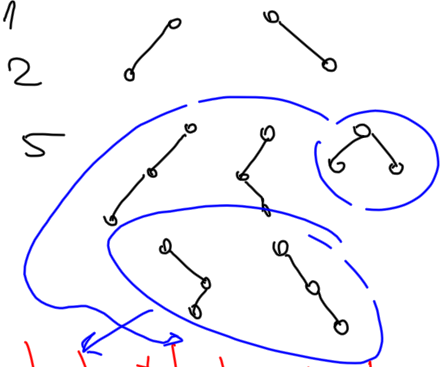
12 12-13:23

$b_0 = 1$

$b_1 = 1$

$b_2 = 2$

$b_3 = 5$



$$b_3 = b_0 \cdot b_2 + b_1 \cdot b_1 + b_2 \cdot b_0$$

12 12-13:28