Důkaz (Věta o setrvačnosti)

$f(x_1,\dots,x_n) = \lambda_1 x_1^2 + \dots + \lambda_r x_r^2,\ \lambda_i \neq 0$

v bázi A (tj. f)

$y_p = \sqrt{\lambda_p}\, x_p$ pro kladné $\qquad y_p = \sqrt{-\lambda_p}\, x_p$ pro záporné

$\Rightarrow \boxed{f(y_1,\dots,y_r) = y_1 + \dots + y_s - y_{s+1} - \dots - y_r}$

$u_1,\dots,u_n$ báze $\qquad = y_1' + \dots + y_{s'}' - y_{s'+1}' - \dots - y_r'$

$P = \langle u_1,\dots,u_s\rangle \qquad Q = \langle v_{s'+1},\dots,v_n\rangle$ v $\tilde{v}$ bázi

$f|_P > 0$ pro nenulové u $\qquad f|_Q \le 0 \Rightarrow P \cap Q = \{0\}$

$\dim P + \dim Q \le n \Rightarrow s + (n - s') \le n \Rightarrow s' \ge s$

opačně $\Rightarrow s \le s' \Rightarrow s = s'$ ✓

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A negativně def. $\Leftrightarrow -A$ je pozitivně def.

Důkaz Lagr. alg.:

- doplnění na čtverce: transformace

$$T = \begin{pmatrix} 1 & -\frac{a_{12}}{a_{11}} & \dots & -\frac{a_{1n}}{a_{11}} \\ 0 & 1 & \dots & 0 \\ \vdots & & & \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

$|A_k| = \lambda_1 \cdots \lambda_k$

A_k, T_k po hlavní minory $\Rightarrow$

$A = T_k^T A_k' (T_k)^{-1} \Rightarrow |A_k| = |A_k'|$

$\Rightarrow$ tedy dostaneme $f(x_1,\dots,x_n) = \lambda_1 x_1^2 + \dots + \lambda_n x_n^2$

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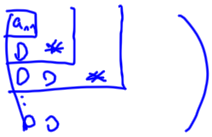

$A = P^T E P \qquad |A| = |P|^2 > 0$

$\Rightarrow$ stačí ukázat pro $|A_k| > 0$.

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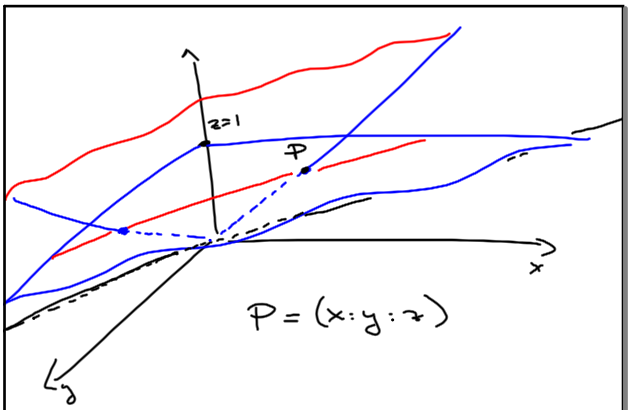

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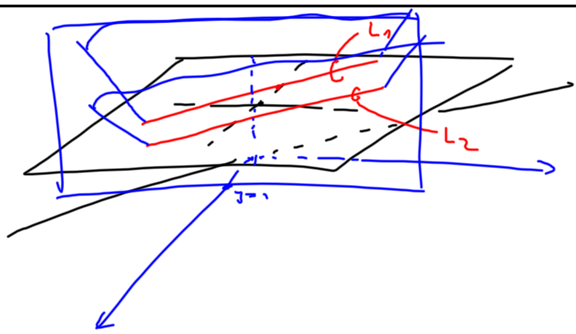

obecně $\quad \mathbb{R}^{n+1} \quad (x_0, x_1, \dots, x_n)$

$\cup$

$A_n \quad (1, x_1, \dots, x_n)$

$\cap$

$\mathbb{R}P_n \quad (x_0 : x_1 : \dots : x_n)$

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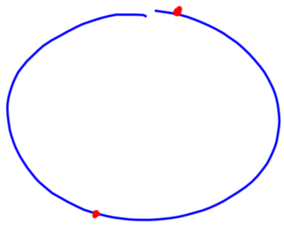

$\{(x_0 : x_1 : x_2 : \dots : x_n)\} \supset \{(0 : x_1 : \dots : x_n)\}$

$\mathbb{R}P_n \qquad \mathbb{R}P_{n-1}$

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$$\begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{pmatrix} \longmapsto A \cdot x$$

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \longmapsto \begin{pmatrix} -f\frac{x}{z} \\ -f\frac{y}{z} \\ 1 \end{pmatrix} \sim \begin{pmatrix} -fx \\ -fy \\ z \end{pmatrix}$$

$$\begin{pmatrix} \quad \end{pmatrix}$$

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$\varphi: \mathbb{RP}_n \to \mathbb{RP}_n$ [illegible] když:

$$\left(\begin{array}{c|ccc} 1 & 0 & \cdots & 0 \\ \hline b_1 & a_{11} & \cdots & a_{1n} \\ \vdots & \vdots & & \\ b_n & a_{n1} & \cdots & a_{nn} \end{array}\right) \begin{pmatrix} 1 \\ x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 1 \\ \\ \\ \end{pmatrix}$$

$$\left(\begin{array}{c|c} 1 & 0 \\ \hline b & A \end{array}\right) \begin{pmatrix} 1 \\ x \end{pmatrix} = \begin{pmatrix} 1 \\ Ax+b \end{pmatrix}$$

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v A_2:

- kružnice $x^2+y^2-1=0$
- parabola $x^2-y=0$
- hyperbola $x^2-y^2-1=0$

v $\mathbb{RP}_2$:

- $x^2+y^2-z^2=0$
- $x^2-yz=0$
- $x^2-y^2-z^2=0$

$\boxed{x^2-y'^2+z'^2}$ ϕ

$y=y'+z'$ nevl. body ϕ

$z=y'-z'$ nebo 1

nebo 2

$x^2+y^2+z^2$ ϕ

$\Leftarrow$ $\boxed{x^2+y^2-z^2} = 0$

$\cancel{x^2-y^2}-z^2$

$-x^2-y^2-z^2$ ϕ

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