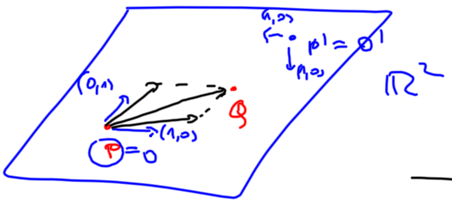
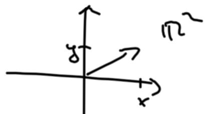




afinní zobrazení
-"- transformace

$v = (x, y)$

10 3-10:03

$(x,y) = x \cdot e_1 + y \cdot e_2$ $\quad F(e_1) = \begin{pmatrix} a \\ c \end{pmatrix}$

$F(x,y) = \underbrace{x \cdot F(e_1) + y \cdot F(e_2)}$ $\quad F(e_2) = \begin{pmatrix} b \\ d \end{pmatrix}$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{matrix} ax + by \\ cx + dy \end{matrix}$$

$$= \underbrace{x\begin{pmatrix} a \\ c \end{pmatrix} + y\begin{pmatrix} b \\ d \end{pmatrix}}$$

skalární součin vektorů u, v $\quad u = \begin{pmatrix} u_x \\ u_y \end{pmatrix}$ $v = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$

$u \cdot v = u_x \cdot v_x + u_y \cdot v_y$

$u^T = (u_x \; u_y) \cdot \begin{pmatrix} v_x \\ v_y \end{pmatrix}$

10 3-10:09

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{matrix} \leftarrow 1 \\ \leftarrow 2 \end{matrix}$ $\quad B = \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix}$, $\begin{matrix} \uparrow & \uparrow \\ 1 & 2 \end{matrix}$

$A \cdot B = (e_{ij})$ $\quad e_{ij} = $ (i-tý řádek) · (j-tý sloupec)

$\boxed{A^{-1}} \cdot A = \begin{pmatrix} 1 & \boxed{0} \\ \boxed{0} & 1 \end{pmatrix}$ KDY EXISTUJE ?

? $(a \; b) \cdot (-b \; a) = 0$ $\quad$ det A

$ax + by = 0$
$cx + dy = 0$

$$\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \boxed{ad-bc} & 0 \\ 0 & ad-bc \end{pmatrix}$$

pokud $\det A \neq 0 \Rightarrow$ $\boxed{A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}}$

10 3-10:12

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \cdot 4 - 3 \cdot 2 = -2 = \det A$$

$$\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} = \det A \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

φ

$$\begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$$

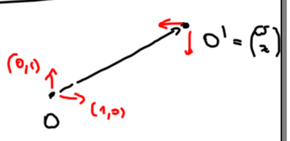
10 3-10:23

$$\begin{pmatrix} x \\ y \end{pmatrix} \xmapsto{\boxed{\Phi}} \boxed{A \cdot \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} w \\ z \end{pmatrix}} = \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$A^{-1} \cdot A \cdot \begin{pmatrix} x \\ y \end{pmatrix} + A^{-1}\begin{pmatrix} w \\ z \end{pmatrix} = A^{-1}\Big(\underbrace{A \cdot \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} w \\ z \end{pmatrix}}_{\begin{matrix} x' \\ y' \end{matrix}}\Big)$$

$$\boxed{\begin{pmatrix} x' \\ y' \end{pmatrix} \mapsto A^{-1}\begin{pmatrix} x' \\ y' \end{pmatrix} - A^{-1}\begin{pmatrix} w \\ z \end{pmatrix}}$$

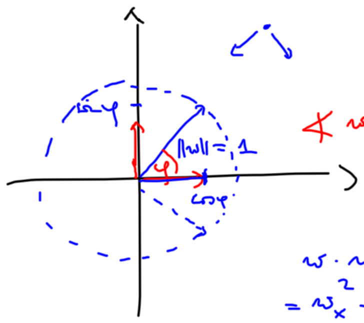
$$= \begin{pmatrix} x \\ y \end{pmatrix}$$



10 3-10:28

$v' = \frac{1}{\|v\|} \cdot v$

$\|v'\| = 1$



$\mathbb{R}^2$

$\measuredangle\, w, (1,0)$

$w \cdot w = \|w\|^2$
$= w_x^2 + w_y^2$

skalární součin $w \cdot v$

$(\alpha w + \beta w') \cdot v$
$= \alpha w \cdot v + \beta w' \cdot v$

10 3-10:36

$v = \begin{pmatrix} v_x \\ v_y \end{pmatrix} \quad s = \begin{pmatrix} s_x \\ s_y \end{pmatrix}$

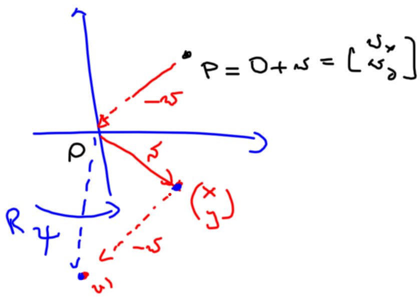
$v' = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \cdot \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} v_x' \\ v_y' \end{pmatrix}$

$s' = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \cdot \begin{pmatrix} s_x \\ s_y \end{pmatrix} = \begin{pmatrix} s_x' \\ s_y' \end{pmatrix}$

$v_x' \cdot s_x' + v_y' \cdot s_y' = \dots$

$= v_x s_x + v_y s_y$

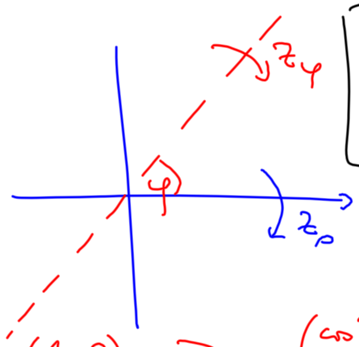
10 3-10:55



$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x - s_x \\ y - s_y \end{pmatrix} \mapsto \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} x - s_x \\ y - s_y \end{pmatrix}$

$\mapsto \begin{pmatrix} \cdot \end{pmatrix} + \begin{pmatrix} s_x \\ s_y \end{pmatrix}$

10 3-10:59



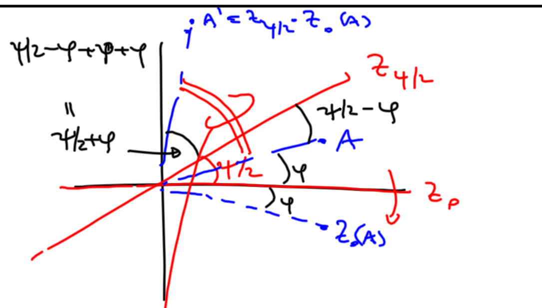
$\cos 2\varphi = \cos^2\varphi - \sin^2\varphi$

$\sin 2\varphi = 2\sin\varphi\cos\varphi$

DŮKAZ

$Z_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad Z_\varphi = \begin{pmatrix} \cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & -\cos 2\varphi \end{pmatrix}$

10 3-11:06



$\pi/2 - \varphi + \pi/2 + \varphi = \pi$

10 3-11:12

$R_{-\alpha} R_{-\beta} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}$

$= \begin{pmatrix} \boxed{\cos\alpha\cos\beta - \sin\alpha\sin\beta} & \cos\alpha\sin\beta + \sin\alpha\cos\beta \\ -\sin\alpha\cos\beta - \cos\alpha\sin\beta & -\sin\alpha\sin\beta + \cos\alpha\cos\beta \end{pmatrix}$

$= \begin{pmatrix} \boxed{\cos(\alpha+\beta)} & -\sin(-\alpha-\beta) \\ \sin(-\alpha-\beta) & \cos(\alpha+\beta) \end{pmatrix}$

10 3-11:20

Důkaz: $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$v = \begin{pmatrix} x \\ y \end{pmatrix} \mapsto A \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix} = A \cdot v$

platí $\|v\| = \|A \cdot v\|$

$x^2 + y^2 = (ax+by)^2 + (cx+dy)^2$

$= \underbrace{(a^2+c^2)}_{=1} x^2 + \underbrace{(b^2+d^2)}_{=1} y^2 + 2\underbrace{(ab+cd)}_{=0} xy$

$\forall x, y \in \mathbb{R}$

$a^2 + c^2 = 1 \quad a = \cos\varphi \quad c = \sin\varphi$

$2 = 1 + 1 + 2 \cdot (\cdot)$

$A = \begin{pmatrix} \cos\varphi & \mp\sin\varphi \\ \sin\varphi & \pm\cos\varphi \end{pmatrix}$

$\begin{vmatrix} \cos\varphi & \mp\sin\varphi \\ \sin\varphi & \pm\cos\varphi \end{vmatrix} = \pm\cos^2\varphi \pm \sin^2\varphi = \pm 1$

$= -1$

10 3-11:24

K ČEMU JE $\det A$?

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ je linearna v vsakem stolpcu

menja znak +/−

$\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} = 0 \quad \begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix} = 0$

$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \quad \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$

$\det(v, w) = \det(v_x e_1 + v_y e_2, w_x e_1 + w_y e_2)$

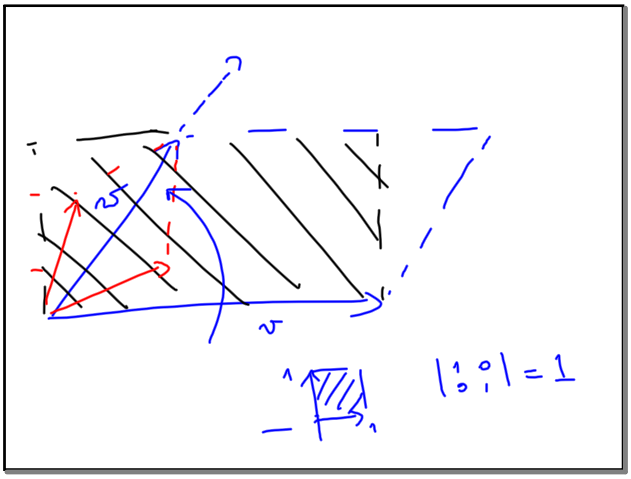
$v = v_x e_1 + v_y e_2$

$w = w_x e_1 + w_y e_2$

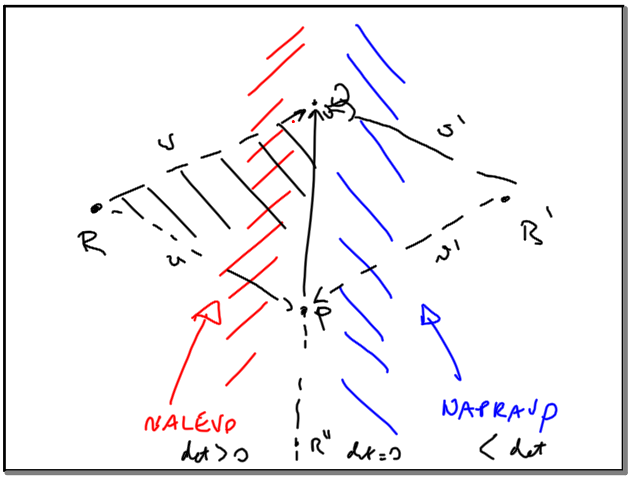
$= \cancel{v_x w_x \det(e_1, e_1)} + v_x w_y \det(e_1, e_2) + v_y w_x \det(e_2, e_1) + \cancel{v_y w_y \det(e_2, e_2)}$

$= (v_x w_y - v_y w_x) \boxed{\det(e_1, e_2)}$

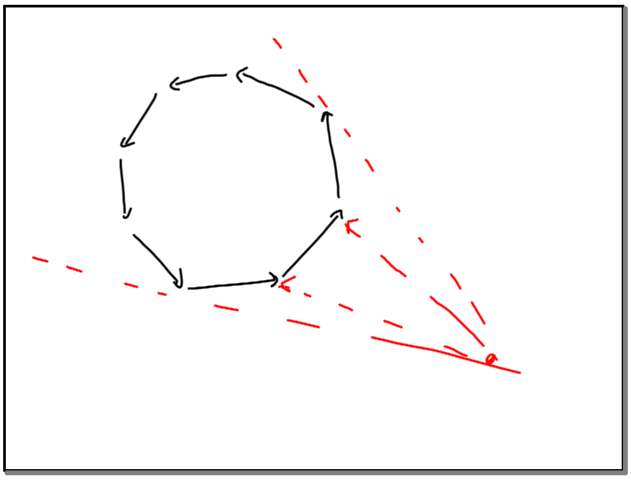
10 3-11:34



10 3-11:40



10 3-11:43



10 3-11:46