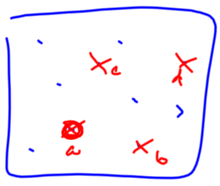
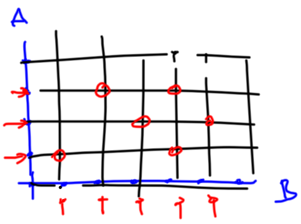


$M \times M \supset R$

kartézský součin

binární .. dvojice



$A \times B \supset R$

10 8-16:04

$\{(a, f(a)),\ a \in A\} \subset A \times B \leftarrow f$

$= R_f$

$\{(b, g(b)),\ b \in B\} \subset B \times C \leftarrow g$

$\{(a,c),\ g(f(a)) = c\}$ $\quad g \circ f$

$= \underbrace{\{(a,c),\ \exists b,\ (a,b) \in f,\ (b,c) \in g\}}_{\varphi}$

10 8-16:24

Hasseho diagram

zobrazení uspořádání na konečné množině

$A = \{a, b, c\}$

$R = \{(c,b), (c,a), (b,a), (a,a), (b,b), (c,c)\}$

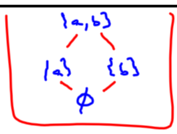
0) $\emptyset$ $\quad R = \emptyset$

1) $A = \{a\}$ $\quad R = \{(a,a)\}$

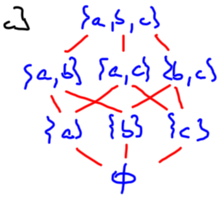
2) $A = \{a, b\}$ $\quad$ · · $\quad R_1 = \{(a,a), (b,b)\}$ $\quad$! $\quad R_2 = \{(a,a), (b,b), (a,b)\}$

10 8-16:35

2^A, $A = \{a, b\}$



$A = \{a, b, c\}$



1) $A \subset A$ ✓ reflexivita

2) $A \subset B$, $B \subset A$ $\Rightarrow A = B$ ✓ antisymetrie

3) $A \subset B \subset C$ ✓ tranzitivita

10 8-16:42

$f \sim g \iff f(0) = g(0)$

1) reflexivita: $f(0) = f(0)$ ✓

2) symetrie: $f(0) = g(0) \Rightarrow g(0) = f(0)$ ✓

3) tranzitivita: $\left.\begin{array}{l} f(0) = g(0) \\ g(0) = h(0) \end{array}\right\} \Rightarrow f(0) = h(0)$ ✓

$f \sim g \iff f(0) = g(1)$

1) $f(0) = f(1)$? NE

2) $f(0) = g(1)$ $\quad g(0) = f(1)$ NE

10 8-16:47

$A = \{3, 4, 5, 6, 7\}$ $\quad$ 1) $a \sim b \iff a \mid b$

$R = \{(3,6), (3,3), (4,4), (5,5), (6,6), (7,7)\}$

je uspořádání? ANO $\quad$ 1) ✓ 2) ✓ 3) ✓

2) $a \sim b \iff a \mid b \vee b \mid a$

$R = \{(3,6), (6,3), (3,3), \dots\}$ ekvivalence

je ekvivalence? ANO

3) $a \sim b \iff$ jsou soudělná

$R = \{(3,3), \dots, (7,7), (3,6), (6,3), (4,6), (6,4)\}$

10 8-17:05

$A = X$, $B = 2^X$

$|A| = 3$ $|B| = 8$

$|A \times B| = |A| \cdot |B| = 24$

kolik různých podmnožin je v $A \times B$

$\Rightarrow 2^{24}$

$|X| = n$ ---> $2^{n \cdot 2^n} = (2^{2^n})^n$

$\boxed{|X| = 3}$

$\boxed{\sum_{i=0}^{n} \binom{n}{i} = 2^n}$

10 8-17:11

A , $\sim_R$ ekvivalence

$[a]$ $[a] = [b] \Leftrightarrow (a,b) \in R$

$\Leftrightarrow (b,a) \in R$

$R_a = \{ b \in A ; (a,b) \in R \}$

$\boxed{R_a \cap R_b \neq \emptyset \Leftrightarrow (a,b) \in R}$

$\Downarrow$ c

$\left.\begin{matrix} a \sim c \\ b \sim c \end{matrix}\right\} \Rightarrow a \sim b \Leftrightarrow R_a = R_b$

10 8-17:17

$\sim_7$ $\boxed{11 \sim_7 25}$

$11 = 1 \cdot 7 + 4$

$25 = 3 \cdot 7 + 4$

$\sim_7$ ekvivalence :

1) reflexivita stejné
2) symetrie opět stejné
3) stejné

$\mathbb{Z}_7 = \{[0], [1], [2], [3], [5], [5], [6]\}$

$(\mathbb{Z}_7, +, \cdot)$?

10 8-17:24

$\boxed{[a] + [b] := [a+b]}$

je to dobře definováno?

$[a] = [a']$, $a' = a + \alpha \cdot 7$ $\alpha, \beta \in \mathbb{Z}$

$[b] = [b']$, $b' = b + \beta \cdot 7$

$[\underbrace{a + \alpha \cdot 7}_{a'} + \underbrace{b + \beta \cdot 7}_{b'}] = [a + b + (\alpha + \beta) \cdot 7]$

$= [a+b]$

$\boxed{[a] \cdot [b] = [a \cdot b]}$

$(a + \alpha \cdot 7)(b + \beta \cdot 7) = a \cdot b + 7(\alpha b + \beta a + 7\alpha\beta)$

10 8-17:27

$\mathbb{Z}_k$ kdy ex. inverze $[a]^{-1}$ $\forall [a] \neq 0$

$\Leftrightarrow k$ je prvočíslo :

$k = p \cdot q$, $p > 1, q > 0$

$\Rightarrow [p] \cdot [q] = [0]$

a nechť ex. $[p]^{-1}$ a $p \cdot q = k$, $[p] \neq 1$

$\Rightarrow [p]^{-1} [p] \cdot [q] = [1] \cdot [q] = [0]$

10 8-17:34

1.107 $|A| = 3$ $|B| = 4$

$f : A \to B$ injekce # ?

1) vybrat obraz $\binom{4}{3}$ možností
2) vybrat permutaci na této množině $3!$

$\Rightarrow \binom{4}{3} \cdot 3!$ možností , tj. 24

1.108 $f : A \to B$ $|A| = 4$ $|B| = 3$

všech 3^4

1) $|\mathrm{Im} f| = 1$ .. # 3
2) $|\mathrm{Im} f| = 2$

10 8-17:40