

$M = (a_1, \dots, a_n)$      $n \cdot v = a_1 b_1 + \dots + a_n b_n$   
 $v = (b_1, \dots, b_n)$

$(A \cdot B) \cdot C = \sum_j a_{ij} \cdot (\sum_k b_{jk} c_{ke}) = c_{ie}$   
 $= \sum_i (\sum_j a_{ij} b_{ji}) c_{ie}$

10 17-10:03

$x = (x_1, \dots, x_n)^T$      $A = (a_{ij})$      $j = 1, \dots, n$   
 $i = 1, \dots, m$

$A \cdot x = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \end{pmatrix}$

$\neq 0$

10 17-10:15

$\begin{pmatrix} \downarrow P \\ \left( \right) \end{pmatrix} \begin{pmatrix} \left( \right) \end{pmatrix} \begin{pmatrix} \downarrow Q \\ \left( \right) \end{pmatrix}$

$A' = P \dots P_1 \cdot A$      $\begin{pmatrix} \text{crossed out} \\ 0 \end{pmatrix}$   
 $A'' = P \dots P_1 \cdot A \cdot Q_1 \dots Q_s$   
 $= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \dots & 0 \\ 0 & 0 & 0 \end{pmatrix}$

10 17-10:25

$A \sim A^{-1} ?$      $(A^{-1} \cdot A = A \cdot A^{-1} = E)$

$(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$      $A \cdot x = b$   
 $(A^{-1} \cdot A) \cdot x = A^{-1} \cdot b$      $x = A^{-1} \cdot b$

$A \mapsto A' = P \cdot A$  (rotations)     $x = A^{-1} \cdot b$   
Wegp. zu und unter GSS

$\Rightarrow (a_{ij}) = A', a_{ij} = 0$      $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$   
 $\neq i=j, a_{ii} \neq 0 \neq i$

$\Rightarrow A'' = P' \cdot P \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$      $\Rightarrow P \cdot P = A^{-1}$

10 17-10:33

$P \cdot A = E = A \cdot Q$

$P \cdot (A|E) = (P \cdot A | P \cdot E) = (E | A^{-1})$

---

$n=2$ :  $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$      $\det A = +ad - bc$   
 $\det A = 0 \Leftrightarrow h(A) < 2$

10 17-10:41

Wk. in der volle n

$1) n=1$  ✓  
 $2) \text{Wegp. mit } n-1.$   
 $x = \{1, \dots, n\}$      $\{0, 1, \dots\}$

$\Rightarrow n(n-1)! \text{ m\u00f6glich s. rekursiv verf.}$

Wegp. zu GSS  
 $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

10 17-11:05

$\frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T) \rightarrow \left( \begin{array}{|c} \text{symmetrisch} \\ \text{antisymmetrisch} \end{array} \right)$

Diagonal det  $A = (a_{ij}) \quad i, j = 1, \dots, n$

2)  $|A| = \sum_{\sigma \in S_n} \text{sgn}(\sigma) a_{1\sigma(1)} \dots a_{n\sigma(n)} = 0$   
 weil für  $j \neq i$   $a_{i\sigma(i)} = 0$

3)  $|B| = \sum_{\sigma \in S_n} \text{sgn}(\sigma) b_{1\sigma(1)} \dots b_{n\sigma(n)} = \sum_{\sigma \in S_n} \text{sgn}(\sigma) a_{1\sigma(1)} \dots a_{j\sigma(j)} \dots a_{i\sigma(i)} \dots a_{n\sigma(n)}$   
 $\dots a_{i\sigma(j)} \dots a_{n\sigma(n)} = - \sum_{\tau \in S_n} \text{sgn}(\tau) a_{1\sigma(1)} \dots a_{j\tau(j)} \dots a_{i\tau(i)} \dots a_{n\tau(n)}$

4)  $n! = 1$

10 17-11:15

5)  $B = \left( \begin{array}{|c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right) \quad C = \left( \begin{array}{|c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right)$   
 $A = \left( \begin{array}{|c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right) = \text{---} + \text{---}$

$\det A = \sum_{\sigma \in S_n} \text{sgn}(\sigma) a_{1\sigma(1)} \dots (b_{i\sigma(i)} + c_{i\sigma(i)}) \dots a_{n\sigma(n)}$   
 $= \sum_{\sigma \in S_n} \text{sgn}(\sigma) a_{1\sigma(1)} \dots b_{i\sigma(i)} \dots a_{n\sigma(n)} = |B|$   
 $+ \sum_{\sigma \in S_n} \text{sgn}(\sigma) a_{1\sigma(1)} \dots c_{i\sigma(i)} \dots a_{n\sigma(n)} = |C|$   
 $\Rightarrow |A| = |B| + |C|$

10 17-11:21

$\sum_{\sigma \in S_n} \text{sgn}(\sigma) a_{1\sigma(1)} \dots a_{n\sigma(n)} = |A|$

$\sum_{\sigma \in S_n} \text{sgn}(\sigma) a_{\sigma(1)1} \dots a_{\sigma(n)n} = |A^T|$   
 $= \sum_{\tau \in S_n} \text{sgn}(\tau) a_{1\tau(1)} \dots a_{n\tau(n)}$

$\text{sgn}(\sigma) = \text{sgn}(\tau) \quad \checkmark$

10 17-11:26

$\left( \begin{array}{|c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right) \quad |A| = a_{11} a_{22} \dots a_{nn}$

10 17-11:30

10 17-11:36

$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 6 & 7 \end{vmatrix} = +1 \begin{vmatrix} 4 & 5 \\ 6 & 7 \end{vmatrix} - 0 \begin{vmatrix} 2 & 3 \\ 1 & 7 \end{vmatrix} + 1 \begin{vmatrix} 2 & 3 \\ 0 & 4 \end{vmatrix}$   
 $= | \begin{smallmatrix} 4 & 5 \\ 6 & 7 \end{smallmatrix} | \cdot 3 - | \begin{smallmatrix} 0 & 5 \\ 1 & 7 \end{smallmatrix} | \cdot 2 + | \begin{smallmatrix} 2 & 3 \\ 6 & 7 \end{smallmatrix} | \cdot 1$

Cauchy  $|A \cdot B| = |A| \cdot |B|$   
 $H = \begin{pmatrix} A & 0 \\ -E & B \end{pmatrix}$

10 17-11:42

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad A \cdot A^* = \begin{pmatrix} d^2 + a^2 & 0 \\ 0 & d^2 + b^2 \end{pmatrix}$$
$$A^* = \begin{pmatrix} d & -b \\ c & a \end{pmatrix}$$

10 17-11:51