důkaz např.:

$$(a+(-a))\cdot u \overset{V2}{=} a\cdot u + (-a)\cdot u \overset{V5}{=} a\cdot u + (-1\cdot(a)\cdot u) \overset{(2)}{=} au - au = 0$$

$$(f+g)(x) = f(x) + g(x)$$

10 24-10:08

Dk. (1) jistě je $\tilde M = \{a_1u_1+\dots+a_ku_k : u_i \in M\}$, $\tilde M \subset \langle M\rangle$. $\tilde M$ je vektorový podprostor. $M \subset \tilde M$

$\Rightarrow \langle M\rangle \subset \tilde M \Rightarrow \tilde M = \langle M\rangle$. ✓

Steinitz: $v=(v_1,\dots,v_n)$ báze V

$u_1,\dots,u_k$ lineárně nezávislé vektory ve V

$\Rightarrow$ lze, čísla $i_1,\dots,i_k$ tž., že vyměníme v_{i_j} za u_j dostaneme opět bázi

Dk. postupně: $u=u_1$: $\boxed{0\neq u} = a_1v_1+\dots+a_nv_n$, $a_1 \neq 0$

$\Rightarrow v_1 = a_1^{-1}(u-a_2v_2-\dots-a_nv_n)$

$\Rightarrow (u,v_2,\dots,v_n)$ opět generuje V, nezávislé.

10 24-10:21

$b_1u+b_2v_2+\dots+b_nv_n = 0$

$b_1 \neq 0$

$\Rightarrow u = b_1^{-1}(-b_2v_2-\dots-b_nv_n)$

$= a_1v_1 + a_2v_2+\dots+a_nv_n$

$a_1 \neq 0$ spor pro $v_2,\dots,v_n$

zbytek indukcí. ∎

10 24-10:39

$W\subset V$

Dk: $W=\langle w_1,\dots,w_k\rangle$ doplníme na bázi

(1)+(2) $V=\langle \boxed{w_1,\dots,w_k}, v_{k+1},\dots,v_n\rangle$

(3): $W_1\cap W_2 = \langle z_1,\dots,z_s\rangle$ báze $\quad \dim(W_1\cap W_2) = s$

$W_1 = \langle z_1,\dots,z_s, w_{s+1},\dots,w_k\rangle \quad \dim W_1 = k$

$W_2 = \langle z_1,\dots,z_s, u_{s+1},\dots,u_r\rangle \quad \dim W_2 = r$

$W_1+W_2 = \langle z_1,\dots,z_s, w_{s+1},\dots,w_k, u_{s+1},\dots,u_r\rangle$

$\dim(W_1+W_2) = s+(k-s)+(r-s)$ lin. nezávislé

$= k+r-s$

10 24-10:42

$$V \xrightarrow{f} W \xrightarrow{g} Z, \qquad g\circ f$$

$(g\circ f)(u) = g(f(u))$

$(g\circ f)(u+v) = g(f(u)+f(v)) = g\circ f(u) + g\circ f(v)$

$f(u) = f(v) \iff u=v$

$f(u-v) = 0 \iff u-v=0$

10 24-10:55

$\begin{pmatrix}1\\0\end{pmatrix}\mapsto\begin{pmatrix}1\\0\end{pmatrix}$ $\qquad \begin{pmatrix}x\\y\end{pmatrix}\mapsto\begin{pmatrix}1&1\\0&0\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$

$\begin{pmatrix}0\\1\end{pmatrix}\mapsto\begin{pmatrix}1\\0\end{pmatrix}$

$m > n$

$V \xrightarrow{u} K^m \xleftarrow{v} W$, $\quad v^{-1}\circ u$ $\qquad K^m \xleftarrow{v} W$

$K^n \ni x \mapsto A\cdot x = y \in K^m$

10 24-11:11

$$\begin{array}{ccccc} V & \xrightarrow{f} & W & \xrightarrow{g} & Z \\ u\downarrow & & \downarrow v & & \downarrow w \\ K^n & \xrightarrow{A} & K^m & \xrightarrow{B} & K^l \end{array}$$

$x \mapsto A\cdot x \mapsto B\cdot(A\cdot x)$

$\qquad\qquad\qquad\qquad\;\; \parallel$

$x \longmapsto (B\cdot A)\cdot x$

$g^{-1} = f$

$x \mapsto A\cdot x \mapsto (B\cdot A)\cdot x = x$

10 24-11:21

$$P\cdot A\cdot Q = \begin{pmatrix} \underbrace{\begin{matrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \\ & 0 & \end{matrix}}_{r(A)} & \begin{matrix} 0 \\ \\ 0 \end{matrix} \end{pmatrix}$$

φ

10 24-11:27

$\boxed{V \longrightarrow K}$

$K = \mathbb{C}$

$z = x + iy \mapsto x - iy = \bar{z}$

lin fun α $\qquad v = (v_1, \dots, v_n) \in V$

$K = \langle 1 \rangle$

$\alpha(u) = (\alpha_1, \alpha_2, \dots, \alpha_n)\cdot\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$

$e_1 = (1\ 0 \dots 0)$

$e_2 = (0\ 1 \dots 0)$

$\vdots$

$V^* = \{\alpha\}$

10 24-11:32

(2) $\Longrightarrow \langle u,u\rangle = \overline{\langle u,u\rangle}$

in $\mathbb{R}$ $\quad |x| = \sqrt{x^2}$

$\mathbb{C}$ $\quad |z|^2 = z\cdot\bar{z}$

$\langle u,u\rangle = \|u\|^2$

$\langle x_1e_1 + \dots + x_ne_n, y_1e_1 + \dots + y_ne_n\rangle =$

$= \sum_{i,j=1}^{n} x_i\cdot\bar{y}_j\langle e_i, e_j\rangle = x_1\bar{y}_1 + \dots + x_n\bar{y}_n$

δ_{ij} $\qquad \langle e_i, e_j\rangle = 0$ for $i \neq j$, 1 for $i = j$

10 24-11:40

$(A\cdot x)^T\cdot\overline{(A\cdot y)} = x^T\cdot\bar{y} = x^T\cdot(A^T\cdot\bar{A})\,\bar{y}$

$f_x^T \qquad f_y \qquad\qquad A^T\cdot\bar{A} = E$

$(A\cdot x)^T\cdot\bar{y} = x^T\cdot\overline{(A\cdot y)}$

$x^T\cdot A^T\cdot\bar{y} = x^T\cdot\bar{A}\,\bar{y}$

$\boxed{A^* = A}$

10 24-11:45