$V \cong (V^*)^*$

$x = (1, 2, i)$
$y = (3, i, 1)$
$\langle x, y \rangle = 1 \cdot 3 + 2(-i) + i \cdot 1 = 3 - i$

$f(v - f(v)) = f(v) - f(f(v)) = f(v) - f(v) = 0$

$u, w \in W^\perp: \langle u + w, v \rangle = \overbrace{\langle u, v \rangle}^{0} + \underbrace{\langle w, v \rangle}_{0} = 0$

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$W^\perp = a_1 u_1 + a_2 u_2 + \dots + a_k u_k = \alpha$ $(a_1, \dots, a_n)$ souřadnice vektoru α $W^\perp$
$(u_1, \dots, u_n)$ ... báze V
Podmínky pro $W^\perp$:
$u = b_1 u_1 + \dots + b_k u_k$

$\left.\begin{array}{l} \langle u_1, \alpha \rangle = 0 \\ \langle u_2, \alpha \rangle = 0 \\ \vdots \\ \langle u_k, \alpha \rangle = 0 \end{array}\right\} \Rightarrow \dim W^\perp = n - k$

$W \cap W^\perp = 0 \Rightarrow V = W \oplus W^\perp$

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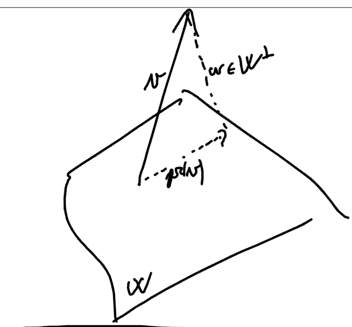

$V = \mathbb{R}^3$

$\overset{\circ}{\scriptstyle o}$ $\langle v, \vec{e}_i \rangle - \langle \vec{e}_i, v \rangle \langle \vec{e}_i, \vec{e}_i \rangle$
$= 0$

$pr(v) = \langle e_1, v \rangle \cdot \vec{e}_1 + \dots + \langle e_k, v \rangle \vec{e}_k$
$\langle v - pr(v), \vec{e}_i \rangle =$
$= \langle v - \langle e_1, v \rangle \cdot \vec{e}_1 - \dots - \langle e_k, v \rangle \vec{e}_k, \vec{e}_i \rangle = \langle v, \vec{e}_i \rangle - \langle e_i, v \rangle$, $1 \le i \le k$

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$\langle f(u+v), f(u+v) \rangle = \langle f(u) + f(v), f(u) + f(v) \rangle =$
$= \langle f(u), f(u) \rangle + \langle f(u), f(v) \rangle + \langle f(v), f(u) \rangle$
$+ \langle f(v), f(v) \rangle$
$\Rightarrow \langle f(v), f(u) \rangle = \frac{1}{2} \big[\langle f(u+v), f(u+v) \rangle$
$- \langle f(u), f(u) \rangle$
$- \langle f(v), f(v) \rangle \big]$
$= \frac{1}{2} \big[\langle u+v, u+v \rangle -$
$- \langle u, u \rangle - \langle v, v \rangle \big]$

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$f: V \to W$ lineární
f prosté $\Leftrightarrow \operatorname{Ker} f = \{\vec{o}\}$:
$f(v_1) = f(v_2) \Rightarrow f(v_1) - f(v_2) = 0$
$= f(v_1 - v_2) = 0 \Rightarrow v_1 - v_2 \in \operatorname{Ker} f$
$\Rightarrow v_1 = v_2$

$Ax =: y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$

$\underbrace{(y_1, \dots, y_n)}_{(Ax)^T} \cdot \begin{pmatrix} \bar{y}_1 \\ \vdots \\ \bar{y}_n \end{pmatrix} = \langle Ax, Ax \rangle$

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$\underset{\langle f(x), f(y) \rangle}{\langle Ax, Ay \rangle} = (Ax)^T \cdot (\bar{A}\bar{y}) = x^T A^T \bar{A} \bar{y} =$
$= x^T A^T \bar{A} \bar{y} = x^T \cdot \bar{y}$
$\langle x, y \rangle$
$\Rightarrow A^T \bar{A} = E$
$\Rightarrow \bar{A}^T A = E$
$\Rightarrow A^{-1} = \bar{A}^T$

$(AB)^T = B^T \cdot A^T$

$\begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \cdot \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}$

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$S^* := \bar{S}^T$

$A = S^{-1} A S$

$$\begin{array}{c|cc} & l & n-l \\ l & \multicolumn{2}{c}{\multirow{2}{*}{}} \\ n-l & & \end{array} \quad \left(\begin{array}{c|c} B & C \\ \hline \underline{0} & D \end{array}\right) \begin{pmatrix} w_1 \\ \vdots \\ w_l \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} B \cdot \begin{pmatrix} w_1 \\ \vdots \\ w_l \end{pmatrix} \\ 0 \cdot \begin{pmatrix} w_1 \\ \vdots \\ w_l \end{pmatrix} \end{pmatrix} \begin{matrix} \}\, l \\ \}\, n-l \end{matrix}$$

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$f(W) \in W \Rightarrow f(W^\perp) \in W^\perp$

$f: V \to V$, unitární

v_1 vlastní vektor $f \Rightarrow \langle v_1 \rangle$ je invariantní vůči f $\left(f(v_1) = \lambda v_1 \in \langle v_1 \rangle \right)$

$\Rightarrow \langle v_1 \rangle^\perp$ je invariantní

uvážeme vl. vektor $f: \langle v_1 \rangle^\perp \to \langle v_1 \rangle^\perp$

nazveme v_2

$\vdots$

$V = \langle v_1 \rangle \oplus \dots \oplus \langle v_n \rangle$

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Nechť v je vlastní vektor příslušný k hodnotě λ $(\notin \mathbb{R})$: $\quad v = (x_1 + iy_1, \dots, x_n + iy_n)$

$v = x + iy = (x_1, \dots, x_n) + i(y_1, \dots, y_n)$

$\lambda = \alpha + i\beta$

$\underline{Ax} + \underline{iAy} = A(x + iy) = \lambda(x + iy) =$

$= (\alpha + \beta i)(x + iy) =$

$= \underline{\alpha x - \beta y} + \underline{\alpha iy + \beta ix}$

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