① Rozhodněte, zda je fce $f(x,y)=\sqrt{|x\cdot y|}$ v bodě $[\underset{x_0}{0},\underset{y_0}{0}]$ diferencovatelná!

$$\exists ? \lim_{t\to 0}\frac{\sqrt{|(x_0+t)(y_0+t)|}-\sqrt{|x_0\cdot y_0|}}{t}=$$

$$=\lim_{t\to 0}\frac{\sqrt{|t^2|}}{t}=\lim_{t\to 0}\frac{|t|}{t}$$

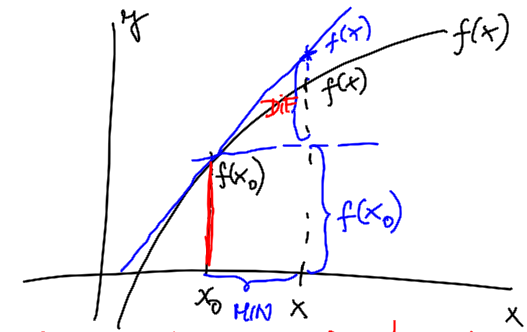
1) je-li $t<0$ pak $\lim_{t\to 0^-}\frac{|t|}{t}=-1$

2) je-li $t>0$ pak $\lim_{t\to 0^+}\frac{|t|}{t}=1$

$\left.\right\}\ \neq \Rightarrow \nexists \lim \Rightarrow \nexists$ derivace

10 8-18:01

# DIFERENCIÁL FUNKCE




$$df(x_0,y_0)=f'_x(x_0,y_0)\underbrace{(x-x_0)}_{dx}+f'_y(x_0,y_0)\underbrace{(y-y_0)}_{dy}$$

10 8-18:10

② Určete diferenciál fce $f(x,y)=\arcsin\frac{x}{\sqrt{x^2+y^2}}$ v bodě $[1,\sqrt{3}]$.

$$f'_x(x,y)=\frac{1}{\sqrt{1-\left(\frac{x}{\sqrt{x^2+y^2}}\right)^2}}\cdot\frac{\sqrt{x^2+y^2}-x\frac{1\cdot 2x}{2\sqrt{x^2+y^2}}}{x^2+y^2}$$

$$f'_y(x,y)=\frac{1}{\sqrt{1-\left(\frac{x}{\sqrt{x^2+y^2}}\right)^2}}\cdot\frac{(-1)x\cdot 2y}{2\sqrt{x^2+y^2}}$$

$$f'_x(1,\sqrt{3})=\frac{1}{\sqrt{1-\frac{1}{1+3}}}\cdot\frac{\sqrt{1+3}-1\cdot\frac{1\cdot\cancel{2}}{\cancel{2}\sqrt{1+3}}}{1+3}=$$

$$=\frac{1}{\sqrt{1-\frac{1}{4}}}\cdot\frac{2-\frac{1}{2}}{4}=$$

$$=\frac{1\cdot 2}{\sqrt{3}}\cdot\frac{\frac{3}{2}}{4}=\frac{\cancel{2}}{\sqrt{3}}\cdot\frac{3}{\cancel{8}4}=\frac{3}{4\sqrt{3}}$$

$$=\frac{\cancel{3}\sqrt{3}}{4\cdot\cancel{3}}=\frac{\sqrt{3}}{4}$$

$$f'_y(1,\sqrt{3})=\frac{1}{\sqrt{1-\frac{1}{4}}}\cdot\frac{(-1)\cdot 2\sqrt{3}}{2\sqrt{1+3}}=$$

$$=\frac{1\cdot\cancel{2}}{\cancel{\sqrt{3}}}\cdot\frac{-\cancel{2}\cancel{\sqrt{3}}}{4\cancel{2}}=\underline{\underline{-1}}$$

$$df(1,\sqrt{3})=\frac{\sqrt{3}}{4}dx-dy$$

$(-1)dy$

10 8-18:16

③a) Pomocí diferenciálu přibližně spočtěte

$\arcsin\frac{0{,}48}{1{,}05}$ $\quad x_0=\frac{1}{2}$, $y_0=1$

$$f(x,y)=\arcsin\frac{x}{y}$$

$$f'_x(x,y)=\frac{1}{\sqrt{1-\frac{x^2}{y^2}}}\cdot\frac{1}{y}$$

$$f'_x(x_0,y_0)=f'_x\left(\tfrac{1}{2},1\right)=\frac{1}{\sqrt{1-\frac{\frac{1}{4}}{1}}}\cdot\frac{1}{1}=$$

$$=\frac{1}{\sqrt{1-\frac{1}{4}}}\cdot 1=\frac{1}{\sqrt{\frac{3}{4}}}=\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$$

$$=\frac{2\sqrt{3}}{3}$$

$$f'_y(x,y)=\frac{1}{\sqrt{1-\frac{x^2}{y^2}}}\cdot x\cdot\left(-\frac{1}{y^2}\right)=$$

$$f'_y\left(\tfrac{1}{2},1\right)=\frac{1}{\sqrt{1-\frac{\frac{1}{4}}{1}}}\cdot\frac{1}{2}\cdot\left(-\frac{1}{1}\right)=$$

$$=\frac{1}{\sqrt{\frac{3}{4}}}\cdot\frac{(-1)}{2}=\frac{\cancel{2}}{\sqrt{3}}\cdot\frac{(-1)}{\cancel{2}}=\frac{-1}{\sqrt{3}}=$$

$$=-\frac{\sqrt{3}}{3}$$

$$f(x_0,y_0)=f\left(\tfrac{1}{2},1\right)=\arcsin\frac{\frac{1}{2}}{1}=$$

$$=\arcsin\frac{1}{2}=\frac{\pi}{6}$$

$$f(0{,}48;1{,}05)\approx\underbrace{\frac{\pi}{6}}_{f(x_0,y_0)}+\frac{2\sqrt{3}}{3}\cdot\left(0{,}48-\tfrac{1}{2}\right)+\left(-\frac{\sqrt{3}}{3}\right)(1{,}05-1)=$$

$$=\frac{\pi}{6}+\frac{2\sqrt{3}}{3}\cdot(-0{,}02)-\frac{\sqrt{3}}{3}\cdot 0{,}05$$

10 8-18:26

# TEČNÁ NADROVINA

(podprostor dim $n-1$ v $n$-rozměrném prostoru)

Mějme fci $z=f(x,y)$, pak tečná rovina v bodě $[x_0,y_0,z_0]$ má obecnou rci

$$\underset{f(x,y)}{z}-\underset{f(x_0,y_0)}{z_0}=\underbrace{f'_x(x_0,y_0)\cdot(x-x_0)}+\underbrace{f'_y(x_0,y_0)(y-y_0)}$$

10 8-18:35

④b) Určete rci tečné nadroviny ke grafu fce $f(x,y)=\operatorname{arctg}\frac{y}{x}$ v $[1,-1,?]$.

$$z_0=f(1,-1)=\operatorname{arctg}\frac{(-1)}{1}=\operatorname{arctg}(-1)=$$

$$=-\frac{\pi}{4}$$

$$f'_x(x,y)=\frac{1}{1+\frac{y^2}{x^2}}\cdot y\left(-\frac{1}{x^2}\right)$$

$$f'_x(1,-1)=\frac{1}{1+1}\cdot(-1)\cdot\left(-\frac{1}{1}\right)=\frac{1}{2}$$

$$f'_y(x,y)=\frac{1}{1+\frac{y^2}{x^2}}\cdot\left(\frac{1}{x}\right)$$

$$f'_y(1,-1)=\frac{1}{1+1}\cdot\left(\frac{1}{1}\right)=\frac{1}{2}$$

$\tau$: $z-\left(-\frac{\pi}{4}\right)=\frac{1}{2}\cdot(x-1)+\frac{1}{2}\cdot(y-(-1))$

$\tau$: $z+\frac{\pi}{4}=\frac{1}{2}x-\cancel{\frac{1}{2}}+\frac{1}{2}y+\cancel{\frac{1}{2}}$

$$\frac{1}{2}x+\frac{1}{2}y-z-\frac{\pi}{4}=0$$

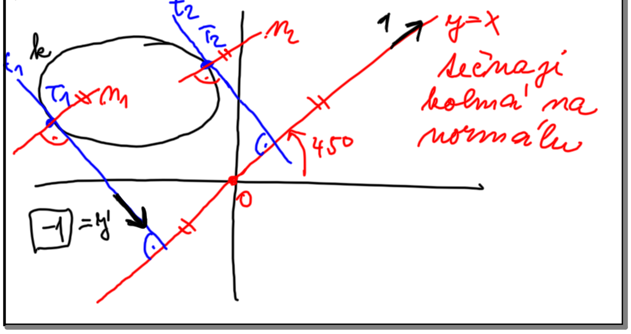
$$\underline{x+y-2z-\frac{\pi}{2}=0}$$

10 8-18:39

10 8-18:45

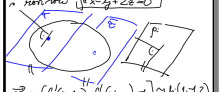
⑤ $k: 3x^2 + 6y^2 - 3x + 3y - 2 = 0$

Najděte body, kdy je normála rovnoběžná s osou 1. kvadrantu.




10 8-18:45

⑥ K elipsoidu o rci $x^2+2y^2+z^2=1$ veďte tečné roviny rovnoběžné s rovinou $\boxed{\rho: x-y+2z=0}$



$$\vec{n}_\tau = \left(f'_x(x_0,y_0), f'_y(x_0,y_0), -1\right) \approx k(1,-1,2)$$

$$\approx \left(-\tfrac{1}{2}, \tfrac{1}{2}, -1\right)$$

$f'_x(x_0,y_0) = -\frac{1}{2} \Rightarrow z'_x(x_0,y_0) = -\frac{1}{2}$

$f'_y(x_0,y_0) = \frac{1}{2} \Rightarrow z'_y(x_0,y_0) = \frac{1}{2}$

Elipsoid je fce daná implicitně, takže vyjádříme $z = f(x,y)$

$z'_x: \quad (x^2+2y^2+z^2=1)'_x$

$2x + 2z \cdot z'_x = 0$

$\cancel{2}z \cdot z'_x = -\cancel{2}x$

$z'_x = -\frac{x}{z}$

$z'_y: \quad (x^2+2y^2+z^2=1)'_y$

$4y + 2z z'_y = 0$

$\cancel{2}z \cdot z'_y = -\cancel{4}^{2}y$

$z'_y = \frac{-2y}{z}$

$-\frac{x}{z} = -\frac{1}{2} \Rightarrow x = \frac{z}{2}$

$-\frac{2y}{z} = \frac{1}{2} \Rightarrow y = -\frac{1 \cdot z}{4}$

Dosadíme do rce elipsoidu

$\left(\frac{z}{2}\right)^2 + 2\left(-\frac{z}{4}\right)^2 + z^2 = 1$

$\frac{z^2}{4} + \cancel{2}\frac{z^2}{\cancel{16}_8} + z^2 = 1$

$\frac{2z^2 + z^2 + 8z^2 - 8}{8} = 0$

$\frac{11z^2 - 8}{8} = 0$

$z^2 = +\frac{8}{11} \Rightarrow z = \pm\sqrt{\frac{8}{11}}$

10 8-18:49

$z = \pm\sqrt{\frac{8}{11}} \qquad z_1 = \sqrt{\frac{8}{11}} \qquad z_2 = -\sqrt{\frac{8}{11}}$

$x = \frac{z}{2} \quad x_1 = \frac{\frac{\sqrt{8}}{\sqrt{11}}}{2} = \frac{\sqrt{2}}{\sqrt{11}} \qquad x_2 = -\frac{\sqrt{2}}{\sqrt{11}}$

$y = -\frac{z}{4} \quad y_1 = \frac{-\frac{\sqrt{8}}{\sqrt{11}}}{4} = \frac{-\sqrt{2}}{2\sqrt{11}} \qquad y_2 = \frac{+\sqrt{2}}{2\sqrt{11}}$

$\tau: \; x - y + 2z + c = 0$

$T_1 = \left[\frac{\sqrt{2}}{\sqrt{11}}, \frac{-\sqrt{2}}{2\sqrt{11}}, \frac{\sqrt{8}}{\sqrt{11}}\right] \in \tau$

$\frac{\sqrt{2}}{\sqrt{11}} + \frac{\sqrt{2}}{2\sqrt{11}} + \frac{2\sqrt{8}}{\sqrt{11}} + c = 0$

$\frac{2\sqrt{2} + \sqrt{2} + 2 \cdot 2 \cdot 2\sqrt{2}}{2\sqrt{11}} + c = 0$

$\frac{11\sqrt{2}}{2\sqrt{11}} + c = 0$

$c = -\frac{11\sqrt{2}}{2\sqrt{11}}$

$\tau: \; x - y + 2z - \frac{11\sqrt{2}}{2\sqrt{11}} = 0$

$T_2 = \left[\frac{-\sqrt{2}}{\sqrt{11}}, \frac{\sqrt{2}}{2\sqrt{11}}, \frac{-\sqrt{8}}{\sqrt{11}}\right] \in \bar{\tau}$

$\frac{-\sqrt{2}}{\sqrt{11}} - \frac{\sqrt{2}}{2\sqrt{11}} + 2\frac{(-\sqrt{8})}{\sqrt{11}} + c = 0$

$\frac{-2\sqrt{11} - \sqrt{2} - 8\sqrt{2}}{2\sqrt{11}} + c = 0$

$c = \frac{11\sqrt{2}}{2\sqrt{11}}$

$\bar{\tau}: \; x - y + 2z + \frac{11\sqrt{2}}{2\sqrt{11}} = 0$

10 8-19:02

# TAYLORŮV POLYNOM

$$T_n(x_0,y_0) = f(x_0,y_0) + f'_x(x_0,y_0)\cdot(x-x_0) + f'_y(x_0,y_0)(y-y_0) + \frac{1}{2!}\left[f''_{xx}(x_0,y_0)(x-x_0)^2 + 2f''_{xy}(x_0,y_0)(x-x_0)(y-y_0) + f''_{yy}(x_0,y_0)(y-y_0)^2\right] + \dots + \frac{1}{n!}\left(\sum_{j=0}^{n}\binom{n}{j}\frac{\partial^n f}{\partial^{n-j}x\,\partial^j y}(x_0,y_0)(x-x_0)^{n-j}(y-y_0)^j\right)$$

10 8-19:04

7b) TP. 2. stupně $[1,1,1]$

$f(x,y,z) = x^{\frac{y}{z}}$

$f(1,1,1) = 1^{\frac{1}{1}} = 1$

$f'_x(x,y,z) = \frac{y}{z}\cdot x^{\left(\frac{y}{z}-1\right)} \Rightarrow f'_x(1,1,1) = 1 \cdot 1^{1-1} = \underline{1}$

$f'_y(x,y,z) = x^{\frac{y}{z}} \cdot \ln x \cdot \frac{1}{z} \Rightarrow f'_y(1,1,1) = 1^1 \ln 1 \cdot \frac{1}{1} = \underline{0}$

$f'_z(x,y,z) = x^{\frac{y}{z}} \ln x \cdot y \cdot \left(-\frac{1}{z^2}\right) \Rightarrow f'_z(1,1,1) = \underline{0}$

$f''_{xx}(x,y,z) = \frac{y}{z}\cdot\left(\frac{y}{z}-1\right)x^{\frac{y}{z}-2} \Rightarrow 1 \cdot 0 \cdot 1^{(-1)} = 0$

$f''_{xy}(x,y,z) = \frac{1}{z}\cdot x^{\frac{y}{z}-1} + \frac{y}{z}\cdot x^{\frac{y}{z}-1}\cdot \ln x \cdot \frac{1}{z}$

$\Rightarrow f''_{xy}(1,1,1) = 1\cdot 1^0 + 1\cdot 1^0 \cdot 0 \cdot 1 = \underline{1}$

$f''_{xz}(x,y,z) = \left(\frac{-y}{z^2}\right)x^{\frac{y}{z}-1} + \frac{y}{z}\cdot x^{\frac{y}{z}-1}\cdot \ln x\left(\frac{-y}{z^2}\right)$

$\Rightarrow f''_{xz}(1,1,1) = (-1)(1^0) + 1 \cdot 1^0 \cdot 0 \cdot (-1) = \underline{-1}$

$f''_{yy}(x,y,z) = \ln x \cdot \frac{1}{z} \cdot x^{\frac{y}{z}} \cdot \ln x \cdot \frac{1}{z}$

$f''_{yy}(1,1,1) = 0$

$f''_{yz}(x,y,z) = \ln x\left[x^{\frac{y}{z}} \cdot \ln x\left(\frac{-y}{z^2}\right)\cdot\frac{1}{z} + x^{\frac{y}{z}}\cdot\left(-\frac{1}{z^2}\right)\right]$

$\Rightarrow f''_{yz}(1,1,1) = \underline{0}$

$f''_{zz} = (\ln x \cdot y)\left[\;\dots\;\right] \Rightarrow 0$

$T_2(1,1,1) = 1 + 1(x-1) + 0(y-1) + 0(z-1) + \frac{1}{2!}\left[0(x-1)^2 + 2\cdot 1\cdot(x-1)(y-1) + 0(y-1)^2 + 2\cdot(-1)\cdot(x-1)(z-1) + 0\cdot(z-1)^2 + 2\cdot 0\cdot(y-1)(z-1)\right]$

10 8-19:15

8) a) Pomocí TP 2. stupně určete $\sin 29^\circ \cdot \operatorname{tg} 46^\circ$

$f(x;y) = \sin x \cdot \operatorname{tg} y$

$[x_0, y_0] = [30^\circ, 45^\circ] = \left[\frac{\pi}{6}, \frac{\pi}{4}\right]$

$f\left(\frac{\pi}{6}, \frac{\pi}{4}\right) = \sin\frac{\pi}{6} \cdot \operatorname{tg}\frac{\pi}{4} = \frac{1}{2} \cdot 1 = \frac{1}{2}$

$f'_x(x,y) = \operatorname{tg} y \cdot \cos x \Rightarrow f'_x\left(\frac{\pi}{6}, \frac{\pi}{4}\right) = \operatorname{tg}\frac{\pi}{4} \cdot \cos\frac{\pi}{6} = 1 \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

$f'_y(x,y) = \sin x \cdot \frac{1}{\cos^2 y} \Rightarrow f'_y\left(\frac{\pi}{6}, \frac{\pi}{4}\right) = \frac{1}{2} \cdot \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} = \frac{1}{\cancel{2}} \cdot \frac{1}{\frac{\cancel{2}}{\cancel{4}\,2}} = \underline{\underline{1}}$

$f''_{xx}(x,y) = \operatorname{tg} y \cdot (-\sin x) \Rightarrow f''_{xx}\left(\frac{\pi}{6}, \frac{\pi}{4}\right) = 1 \cdot \left(-\frac{1}{2}\right) = -\frac{1}{2}$

$f''_{xy}(x,y) = \frac{1}{\cos^2 y}(\cos x) \Rightarrow f''_{xy}\left(\frac{\pi}{6}, \frac{\pi}{4}\right) = \frac{\frac{\sqrt{3}}{2}}{\left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\frac{\sqrt{3}}{\cancel{2}}}{\frac{\cancel{2}}{\cancel{4}\,2}} = \sqrt{3}$

$f''_{yy}(x,y) = \left(\frac{\sin x}{\cos^2 y}\right)'_y = \sin x \cdot \frac{1(-2)}{\cos^3 y} \cdot (-\sin y)$

$\Rightarrow f''_{yy}\left(\frac{\pi}{6}, \frac{\pi}{4}\right) = \frac{1}{\cancel{2}} \cdot \frac{(\cancel{+2}) \cdot \cancel{\left(\frac{\sqrt{2}}{2}\right)}}{\left(\frac{\sqrt{2}}{2}\right)^{\cancel{3}\,2}} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} = \frac{1}{\frac{2}{4}} = \underline{\underline{2}}$

$$T_2(x_0, y_0) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(-\frac{\pi}{180}\right) + 1\left(\frac{\pi}{180}\right) + \frac{1}{2!}\left(-\frac{1}{2}\left(-\frac{\pi}{180}\right)^2 + 2\sqrt{3}\left(-\frac{\pi}{180}\right)\left(\frac{\pi}{180}\right) + 2\left(\frac{\pi}{180}\right)^2\right) =$$

$$= \frac{1}{2} - \frac{\sqrt{3}}{360}\pi + \frac{\pi}{180} - \frac{\pi^2}{4 \cdot 180^2} - \sqrt{3}\left(\frac{\pi^2}{180^2}\right) + \frac{\pi^2}{180^2}$$

10 8-19:29