1) $f(x,y) = (x^2y+y)^4$

$f'_x(x,y) = 4(x^2y+y)^3 \cdot 2y \cdot x$

$f'_y(x,y) = 4(x^2y+y)^3 \cdot (x^2+1)$

2) $f(x,y) = e^{\frac{x}{y}} + x^{y-1}$ $\qquad \frac{x}{y} = x \cdot y^{-1}$

$f'_x(x,y) = \frac{1}{y} e^{\frac{x}{y}} + (y-1) \cdot x^{y-2}$

$f'_y(x,y) = -e^{\frac{x}{y}} \cdot x \cdot y^{-2} + (\ln x) x^{y-1}$

3) $f(x,y,z) = \left(\frac{x}{y}\right)^z$

$f'_x(x,y,z) = z \cdot \left(\frac{x}{y}\right)^{z-1} \cdot \frac{1}{y}$

$f'_y(x,y,z) = -z \left(\frac{x}{y}\right)^{z-1} \cdot x \cdot y^{-2}$

$f'_z(x,y,z) = \left(\frac{x}{y}\right)^z \cdot \ln\left(\frac{x}{y}\right)$

10 1-17:59

1) $f(x,y) = \ln\left(x + \sqrt{x^2+y^2}\right)$, $A[1,2]$ $\qquad \sqrt{x^2+y^2} = (x^2+y^2)^{\frac{1}{2}}$

$f'_x(x,y) = \frac{1}{x+\sqrt{x^2+y^2}} \cdot \left(1 + \frac{1}{2} \cdot \frac{2x}{\sqrt{x^2+y^2}}\right)$

$f'_x(1,2) = \frac{1}{1+\sqrt{5}} \cdot \left(1 + \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{5}}\right) =$

$= \frac{1}{1+\sqrt{5}}\left(1 + \frac{\sqrt{5}}{5}\right) =$

$= \frac{1}{1+\sqrt{5}} \cdot \left(\frac{5+\sqrt{5}}{5}\right)$

$f'_y(x,y) = \frac{1}{x+\sqrt{x^2+y^2}} \cdot \frac{2y}{\sqrt{x^2+y^2}}$

10 1-18:12

2. $f(x,y) = (1+\log_y x)^3$ $\qquad A[e,e]$

$\log_y x = \frac{\ln x}{\ln y}$

$f(x,y) = \left(1 + \frac{\ln x}{\ln y}\right)^3$ $\qquad \ln x \cdot \ln^{-1} y$

$f'_x(x,y) = 3 \cdot \left(1 + \frac{\ln x}{\ln y}\right)^2 \cdot \left(\frac{1}{x \ln y}\right)$

$f'_y(x,y) = 3 \cdot \left(1 + \frac{\ln x}{\ln y}\right)^2 \cdot \left(-\ln x \cdot \frac{1}{\ln^2 y} \cdot \frac{1}{y}\right)$

$\left((\ln y)^{-1}\right)' = -1 \cdot (\ln y)^{-2} \cdot \frac{1}{y}$

$= -\frac{1}{y \ln^2 y}$

$f(x,y,z) = \operatorname{arctg}\sqrt{xy} + z^z$ $\qquad (xy)^{\frac{1}{2}}$ $\quad x^{\frac{1}{2}y}$

$f'_x(x,y,z) = \frac{1}{1+xy} \cdot \frac{1}{2}y \cdot x^{\frac{1}{2}y-1}$

$f'_y(x,y,z) = \frac{1}{1+xy} \cdot \ln x \cdot x^{\frac{1}{2}y} \cdot \frac{1}{2}$

$f'_z(x,y,z) = z^z \cdot (\ln z + 1)$

∇ $\boxed{z^z} = e^{\ln z^z} = \boxed{e^{z \cdot \ln z}}$ $!$

10 1-18:16

$f(x,y) = x \cdot \ln xy$

f'''_{xyy}

$f'_x(x,y) = \ln xy + x \cdot \frac{1}{xy} \cdot y$

$= \ln xy + 1$

$f''_{xy}(x,y) = \frac{1}{xy} \cdot x = \frac{1}{y}$

$f'''_{xyy} = -\frac{1}{y^2}$

10 1-18:26

$f(x,y) = \ln(1+x+y)$

$\frac{\partial^{136} f}{\partial^{79} x\, \partial^{57} y} = \frac{\partial^{136} f}{\partial^{136} x}$

$f'_x = \frac{1}{1+x+y} \cdot 1 = (1+x+y)^{-1}$

$f''_{xx} = \frac{-1}{(1+x+y)^2} = -(1+x+y)^{-2}$

$f'''_{xxx} = \frac{-(-2)}{(1+x+y)^3} = 2 \cdot (1+x+y)^{-3}$

$f''''_{xxxx} = \frac{-6}{(1+x+y)^4}$

$\boxed{f^{(k)}_{kx} = \frac{(-1)^{k-1} \cdot (k-1)!}{(1+x+y)^k}}$

MI: 1) $k=1$ ✓

2) IP ... Nechť tvrzení platí pro $k=n$

3) $k=n+1$

$f^{(n+1)}_{(n+1)x}(x,y) = \left(f^{(n)}_{nx}(x,y)\right)'_x =$

$= \left(\frac{(-1)^{n-1}(n-1)!}{(1+x+y)^n}\right)'_x =$

$= \left((-1)^{n-1} \cdot (n-1)! \cdot (1+x+y)^{-n}\right)'_x =$

$= (-1)^{n-1}(n-1)!(-n) \cdot (1+x+y)^{-n-1} =$

$= \frac{(-1)^n \cdot n!}{(1+x+y)^{n+1}}$

$f^{(136)}_{136x} = \frac{-135!}{(1+x+y)^{136}}$

10 1-18:30

$f(x,y) = \ln\left(x + \frac{1}{y}\right)$

$\operatorname{grad} f = (f'_x, f'_y)$ $\qquad \left(1, -\frac{16}{9}\right)$

$f'_x = \frac{1}{x+\frac{1}{y}} = \frac{1}{\frac{xy+1}{y}} = \frac{y}{xy+1}$

$f'_y = \frac{-1}{\left(x+\frac{1}{y}\right)y^2} = \frac{-1}{xy^2+y} =$

$= \frac{-1}{y(xy+1)}$

$\operatorname{grad} f = \left(\frac{y}{xy+1}, \frac{-1}{y(xy+1)}\right)$

$\frac{y}{xy+1} = 1$ $\qquad \frac{-1}{y(xy+1)} = -\frac{16}{9}$

$\frac{-y}{y \cdot y(xy+1)} =$

$= -\left(\frac{y}{xy+1}\right) \cdot \frac{1}{y^2}$

$\Downarrow$

$-\frac{1}{y^2} = -\frac{16}{9}$

$y^2 = \frac{9}{16}$

$y = \pm\frac{3}{4}$

$\frac{y}{xy+1} = 1$

i) $y = \frac{3}{4}$ $\qquad$ ii) $y = -\frac{3}{4}$ ...

$\frac{\frac{3}{4}}{\frac{3}{4}x+1} = 1 \Rightarrow \frac{3}{4} = \frac{3}{4}x + 1$

$-\frac{1}{4} = \frac{3}{4}x$

$x = -\frac{1}{3}$

10 1-18:27

$f(x,y) = (x^2+y^2)^{\frac{3}{2}}$

$|\text{grad} f| = 2$

$f'_x = \frac{3}{2}\sqrt{x^2+y^2} \cdot 2x$

$f'_y = \frac{3}{2}\sqrt{x^2+y^2} \cdot 2y$

$\text{grad} f = \left(\frac{3}{2}\sqrt{x^2+y^2}\, 2x, \frac{3}{2}\sqrt{x^2+y^2} \cdot 2y\right)$

$|\text{grad} f| = \sqrt{f_x'^2 + f_y'^2}$

$|\text{grad} f| = \sqrt{\frac{9}{4} \cdot 4x^2 \cdot (x^2+y^2) + \frac{9}{4} 4y^2 \cdot (x^2+y^2)} =$

$|\text{grad} f| = \sqrt{(x^2+y^2)(9x^2+9y^2)}$

$|\text{grad} f| = \sqrt{9 \cdot (x^2+y^2)^2} = 3(x^2+y^2)$

$3(x^2+y^2) = 2$

$x^2+y^2 = \frac{2}{3}$

10 1-18:49

$f'_{\vec{u}} = \lim_{t \to 0} \frac{f(A+tu) - f(A)}{t}$

$g = f(A+tu)$

$g'(0)$ $\quad \|u\| = 1$

$= (\text{grad } f)(A) \cdot (u)$

$f(x,y) = x^4 \cdot \text{arctg}\, xy$

$A[2,0]$

i) $f_x(x,y) = 4x^3 \cdot \text{arctg}\, xy + x^4 \frac{y}{1+x^2y^2}$

ii) $f_y(x,y) = x^4 \frac{x}{1+x^2y^2} = \frac{x^5}{1+x^2y^2}$

iii) $f_x(2,0) = 0$

$f_y(2,0) = \frac{32}{1+0} = 32$

$(\text{grad} f)(A) = (0,32)$

2) $u = (3,4)$

$\|u\| = 5$

$\vec{u} = \left(\frac{3}{5}, \frac{4}{5}\right)$

$f'_{\vec{u}}(A) = (0,32) \cdot \left(\frac{3}{5}, \frac{4}{5}\right) = \frac{128}{5}$

$(0,32)(0,1) = 32$

10 1-18:56

$f(x,y) = \frac{x^2-y^2}{x^2+y^2}$

$A = [1,1]$

$u = (2,1)$

$g(t) = f(A + t \cdot u) = f(1+2t, 1+t)$

$g(t) = \frac{(1+2t)^2 - (1+t)^2}{(1+2t)^2 + (1+t)^2} =$

$= \frac{1+4t+4t^2-1-2t-t^2}{1+4t+4t^2+1+2t+t^2}$

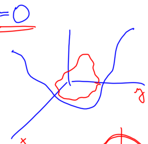
$= \frac{3t^2+2t}{5t^2+6t+2}$

$g'(t) = \frac{(6t+2)(5t^2+6t+2) - (3t^2+2t)(10t+6)}{(5t^2+6t+2)^2}$

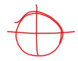
$g'(0) = \frac{2 \cdot 2}{2 \cdot 2} = 1$

10 1-19:07

$F(x,y) = 0$



$x^2+y^2-1 = 0$



$y = f(x)$

$2x + 2 \cdot y \cdot y' = 0$

$y' = \frac{-2x}{2y}$

$y' = -\frac{x}{y}$

$y' = -\frac{F_x(x,y)}{F_y(x,y)}$

$F(x,y) = x^2+y^2-1$

$F'_x(x,y) = 2x$

$F'_y(x,y) = 2y$

$y' = -\frac{2x}{2y} = -\frac{x}{y}$

10 1-19:15

$5x^2+3xy-2y^2+2 = 0$

$y = f(x)$

$10x + 3 \cdot y + 3xy' - 4y \cdot y' = 0$

$y'(3x-4y) = -10x-3y$

$y' = \frac{10x+3y}{4y-3x}$

$\sqrt[x]{y} - \sqrt[y]{x} = 0$

$\sqrt[x]{y} = \sqrt[y]{x} \quad / \cdot \ln$

$\ln y^{\frac{1}{x}} = \ln x^{\frac{1}{y}}$

$\frac{1}{x} \cdot \ln y = \frac{1}{y} \ln x$

$y \cdot \ln y = x \ln x$

$y \ln y - x \ln x = 0$

$F(x,y) = y \cdot \ln y - x \ln x$

$F'_x(x,y) = -\ln x - 1$

$F'_y(x,y) = \ln y + 1$

$y' = -\frac{F_x}{F_y}$

$= \frac{\ln x + 1}{\ln y + 1}$

10 1-19:20

$y = \sqrt{\frac{(x+1)^3}{x-1}}$ $\quad /^2$

$y^2 = \frac{(x+1)^3}{x-1}$

$y^2(x-1) = (x+1)^3$

$xy^2 - y^2 = (x+1)^3$

$xy^2 - y^2 - (x+1)^3 = 0$

$y^2 + x \cdot 2y \cdot y' - 2yy' - 3(x+1)^2 = 0$

$y'(2xy - 2y) = 3(x+1)^2 - y^2$

$y' = \frac{3(x+1)^2 - y^2}{2xy - 2y}$

$y = \sqrt{\bigcirc}$

$y' = \frac{3(x+1)^2 - \frac{(x+1)^3}{x-1}}{2\sqrt{\frac{(x+1)^3}{x-1}}\,(x-1)}$

10 1-19:26

$2x^3+2y^3-4xy=0$

$\underline{T[1,1]}$

$y-y_0=y'(x-x_0)$

$y-1=y'(x-1)$

$6x^2+6y^2y'-4y-4xy'=0$

$y'(6y^2-4x)=4y-6x^2$

$\boxed{y'=\dfrac{4y-6x^2}{6y^2-4x}}$

$y'_{(1,1)}=\dfrac{-2}{2}=\underline{-1}$

$y-1=-1(x-1)$

$\underline{y+x-2=0}$

$\underline{y''}=\dfrac{(4y'-12x)(6y^2-4x)-(4y-6x^2)(12yy'-4)}{(6y^2-4x)^2}$

$\underline{y''_{(1,1)}}=\dfrac{(4\cdot(-1)-12\cdot1)(6-4)-(-2)(-16)}{4}$

$=<0$

10 1-19:31

$x^2+3y^2-2x+6y-8=0$

(

$t \parallel x$

$y=\underline{k}x+q$

$\underline{k=0}$

$\underline{y'_{(x_0,y_0)}=0}$

$2x+6yy'-2+6y'=0$

$y'(6y+6)=2-2x$

$y'=\dfrac{2-2x}{6y+6}$

$\dfrac{2-2x_0}{6y_0+6}=0 \Rightarrow \underline{x_0=1}$

$x_0^2+3y_0^2-2x_0+6y_0-8=0$

10 1-19:39

10 1-19:34