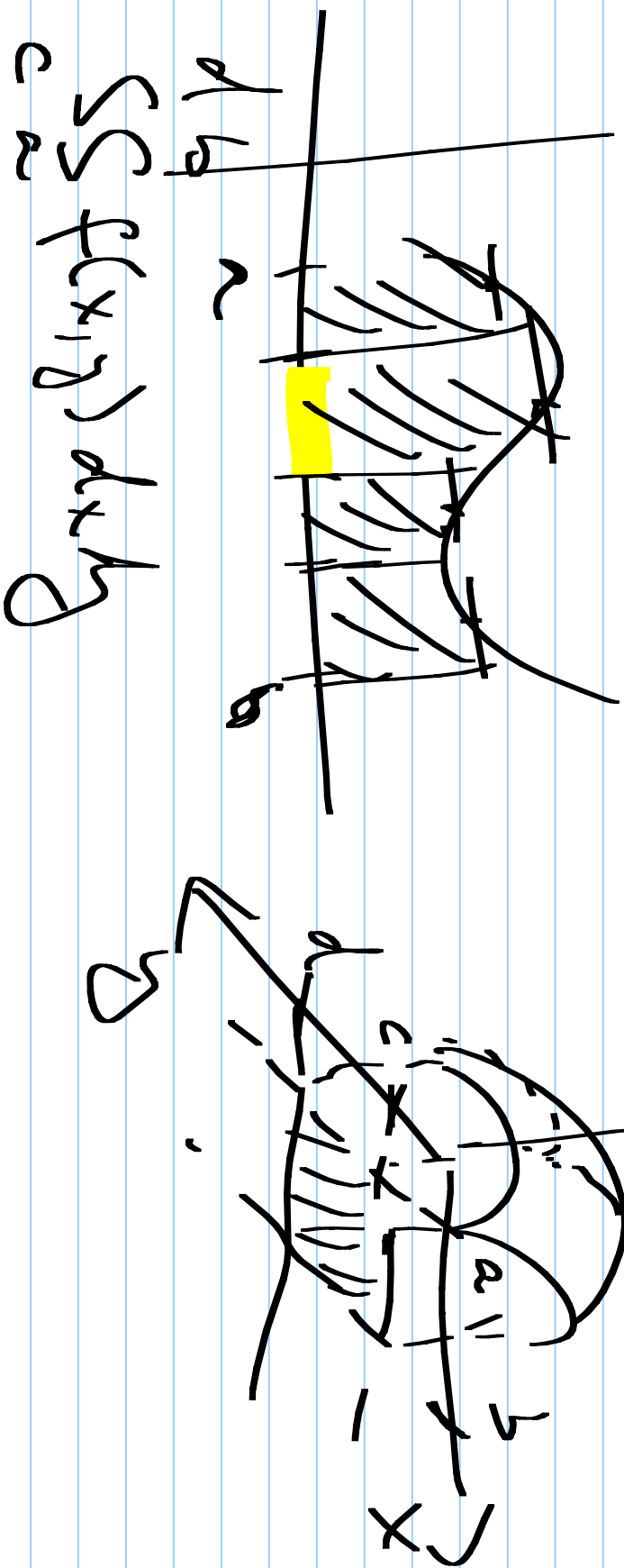
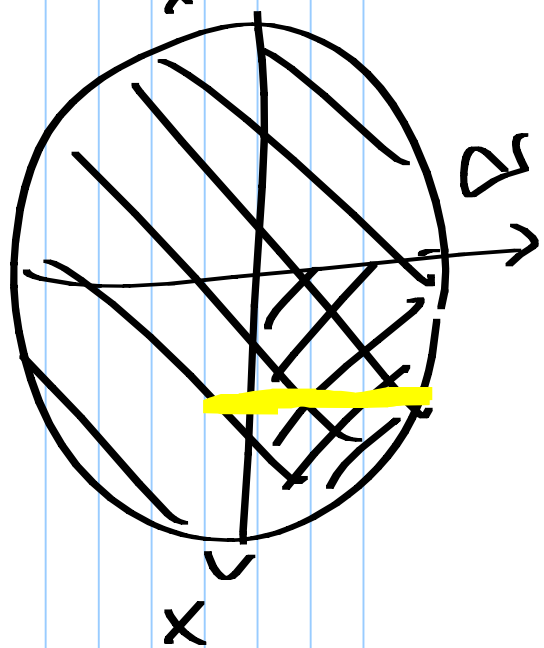


$$\int_a^b f(x) dx = F(b) - F(a), \quad F'(x) = f(x)$$



$$\text{plan } \{x^2 + y^2 \leq 1\} = M$$



$$S = \int_M 1 \, dx \, dy = 4 \int_0^1 \int_0^{\sqrt{1-x^2}} dy \, dx$$

$$= 4 \int_0^1 \sqrt{1-x^2} \, dx = \pi$$

$$\Phi \begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases} \quad D^1 \Phi: \begin{pmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{pmatrix} \quad |D^1 \Phi| = r(\cos^2 \theta + \sin^2 \theta) = r$$

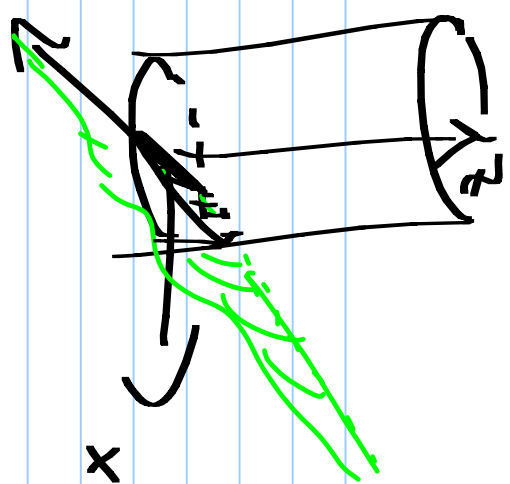
$$M = \{r \leq 1\} \quad S = \int_0^{2\pi} \int_0^1 r \, dr \, d\theta = 2\pi \int_0^1 r \, dr = \pi$$

✓

8.69 a) $\text{with } x^2 + y^2 \leq 1 \text{ and } z \geq 0$

with $z \in x$

with $z > 0$



$x = r \cos \theta$

$M = \{ r \leq 1, z \geq 0 \}$

$\vartheta = r \sin \theta$

$z \leq r \cos \theta$

$\vec{\Phi} = \begin{pmatrix} \sqrt{1-x^2} \\ 0 \\ 0 \\ 1 \end{pmatrix}$

$V = \int_{-\pi/2}^{\pi/2} \int_{0-\pi/2}^{\pi/2} \int_{0-\pi/2}^{\pi/2} r \, dr \, d\theta \, dz = \int_{-\pi/2}^{\pi/2} \int_{0-\pi/2}^{\pi/2} r^2 \cos \theta \, d\theta \, dr = 2 \int_0^1 r^2 \, dr = \frac{2}{3}$