Nový nadpis 18.9.2013

$$f(x,y,z) = \sqrt{xy\ln z}$$

$$f_x = \frac{1}{2}\frac{y\ln z}{\sqrt{xy\ln z}} \qquad f_y = \frac{1}{2}\frac{x\ln z}{\sqrt{xy\ln z}} \qquad f_z = \frac{1}{2}\frac{xy}{z\sqrt{xy\ln z}}$$

$$f_{xz} = \frac{1}{2}\frac{\frac{1}{2}y\sqrt{xy\ln z} - y\ln z\,\frac{1}{2}\frac{xy\frac{1}{z}}{\sqrt{xy\ln z}}}{xy\ln z} = \frac{1}{2}\frac{\sqrt{xy\ln z} - \frac{1}{2}\frac{yx\ln z}{\sqrt{xy\ln z}}}{xz\ln z}$$

$$f_{zx} =$$

Define: $f(x,y): E_2 \to \mathbb{R}$

$$f_{xy}(x,y) = \lim_{t \to 0} \frac{1}{t}\left(f_x(x,y+t) - f_x(x,y)\right)$$

$$= \lim_{t \to 0} \frac{1}{t}\left(\lim_{s \to 0} \frac{1}{s}\left(f(x+s,y+t) - f(x,y+t) - f(x+s,y) + f(x,y)\right)\right)$$

now take: $f_{xy}(x,y) = \lim \frac{1}{t^2}\Big(\underbrace{f(x+t,y+t) - \ldots}_{\varphi(x,y,t)}\Big)$

we prove t $\underbrace{g(x,y) = f(x+t,y) - f(x,y)}$

$$g(x, y+t) - g(x, y) \approx t \cdot g_y(x, y+t_0)$$

$$g_y = f_y(x+t, y) - f_y(x, y)$$

$$\varphi(x, y, t) = \frac{1}{t} g_y(x, y+t_0)$$

$$= \frac{1}{t}\left(f_y(x+t, y+t_0) - f_y(x, y+t_0)\right)$$

$$\varphi(x, y, t) \overset{t \to 0}{\longrightarrow} f_{yx}(x+t_0, y+t_0) \overset{t \to 0}{\longrightarrow} f_{yx}(x, y)$$

$$f_{xy}(x, y) = f_{yx}(x, y)$$

$$(x(t), y(t)) = (x_0 + t\xi, \; y_0 + t\eta)$$

$$\big(f(x(t), y(t))\big)' = f_x(x, y)\, x'(t) + f_y(x, y) \cdot y'(t)$$

$$\big(f(x(t), y(t))\big)''\Big|_{t=0} = f_{xx}(x_0, y_0) \cdot \xi^2 + f_{xy}(x_0, y_0)\eta\xi + f_{yx}(x_0, y_0)\eta\xi + f_{yy}(x_0, y_0)\eta^2$$

$f: E_2 \to \mathbb{R}$ $\quad f(x,y) = xy\cos y \quad P=[\pi,\pi]$

$B=[0,\pi/2,0]$

$$T_2 f(x,y) = f(\pi,\pi) + f_x(\pi,\pi)(x-\pi) + f_y(\pi,\pi)(y-\pi) + \dots$$

$$= -\pi^2 + (y\cos y)\big|_P (x-\pi) + (x\cos y - xy\sin y)\big|_P (y-\pi)$$

$$+\tfrac{1}{2}\Big(0 + 2(\cos y - y\sin y)\big|_P (x-\pi)(y-\pi) + (-x\sin y - x\sin y - xy\cos y)\big|_P (y-\pi)^2\Big)$$

$$= -\pi^2 - \pi(x-\pi) - \pi(y-\pi) - 2(x-\pi)(y-\pi) + \tfrac{1}{2}\pi^2(y-\pi)^2 = \boxed{\tfrac{1}{2}\pi^4 - y\pi^3} - xy + \tfrac{1}{2}\pi^2 y^2$$

"z"

$$f(x,y) = x^2y + y^2x - xy$$

extrema?

$$f_x = 2xy + y^2 - y = 0$$

$$f_y = x^2 + 2yx - x = 0$$

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} \cdot 1 \cdot \begin{vmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{vmatrix} = 7/9$$

$[0,0]$ $(1,0)$ $[0,1]$ $[1/3, 1/3]$ ✓