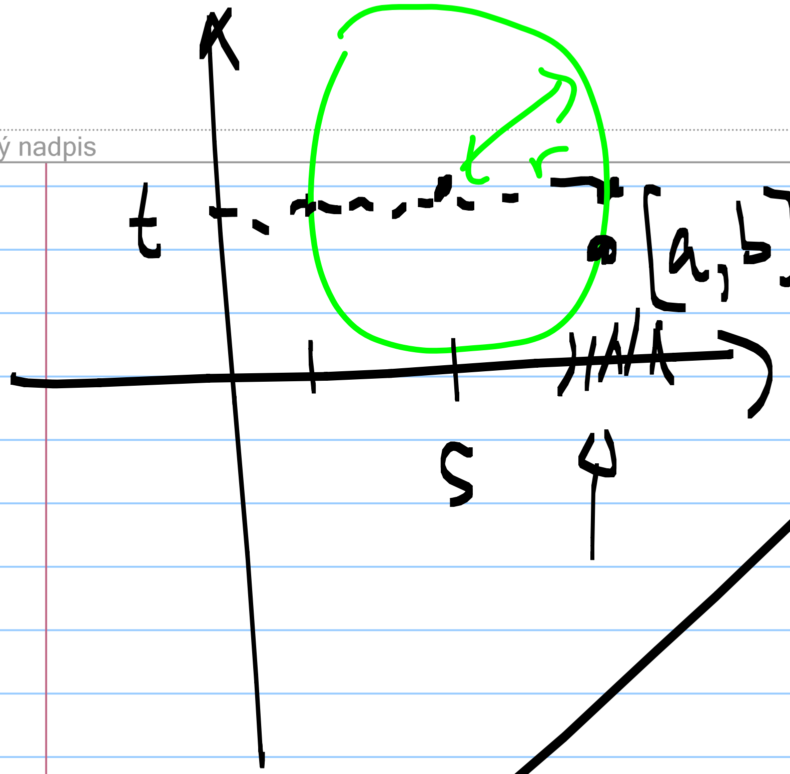

$y = f(x)$ $\quad x, y \in \mathbb{R}^1$

$$\boxed{y - f(x) = F(x,y) = 0}$$

$D^1_x F$ invert. $\quad x = g(y)$

$= -D^1 f \quad D^1 g = +(D^1 f)^{-1} \cdot D_y F \in E_n$

$= +(D^1 f)^{-1}$

Bsp. (Umk., inv. Abb.) $F: \mathbb{R}^n \to \mathbb{R}^n$

$$E_n = D^1(F^{-1} \circ F)(x) = \underbrace{(D^1 F^{-1})(F(x))}_{= (D^1 F)^{-1}} \cdot \underbrace{D^1 F(x)}$$

?

$F(x_0) = y_0 \quad F(x) = y$

$$\frac{1}{\|y - y_0\|}\left(x - x_0 - (D^1 F)^{-1}(y_0)(y - y_0)\right) \to 0$$

$$F(x) - F(x_0) - D^1F(x_0)(x - x_0) = \alpha(x - x_0)$$

$$\frac{1}{\|x - x_0\|}\alpha(x - x_0) \to 0 \quad \text{für } x \to x_0$$

$$\lim_{y \to y_0} \frac{1}{\|y-y_0\|} \left(x - x_0 - \left(D^1F(x_0)\right)^{-1} \left(D^1F(x_0)(x-x_0) + \alpha(x-x_0) \right) \right\|$$

$$= \lim_{y \to y_0} \frac{-1}{\|y-y_0\|} \left(\left(D^1F(x_0)\right)^{-1} \left(\alpha(x-x_0) \right) \right)$$

$$= \left(D^1F(x_0)\right)^{-1} \lim_{y \to y_0} \frac{-1}{\|y-y_0\|} \alpha(x-x_0)$$

$$\|F(x) - F(x_0)\| \geq C\|x - x_0\|$$

$$C^{-1}\|y - y_0\| \geq \|F^{-1}(y) - F^{-1}(y_0)\|$$

$F:\mathbb{R}^n \to \mathbb{R}^n$ $\quad G:\mathbb{R}^n \to \mathbb{R}^n$ לינארי

$\tilde{F} = G \circ F \qquad D^1\tilde{F}(x_0) = E_n$ $\quad G = (D^1F(x_0))^{-1}$

$K(x) = \tilde{F}(x) - x$ [illegible] x_0 $\quad D^1K = 0$

$$\boxed{\|K(x) - K(y)\| \leq C\sqrt{n}\,\|x-y\|}$$

[illegible]

x_0

$\|K(x) - K(y)\| \leq \frac{1}{2}\|x-y\|$

$$\underline{\|K(x) - K(y)\|} \le \frac{1}{2}\|x-y\| \qquad \underline{K(x)} = F(x) \underline{-x}$$

$$\|\underbrace{(u-v)+v}_{u}\| \le \|u-v\| + \|v\|$$

$$\|u\| - \|v\| \le \|u-v\|$$

$$\|y-x\| - \|F(x) - F(y)\| \le \|F(x) - F(y) + y - x\|$$

$$\le \frac{1}{2}\|y-x\|$$

$$\boxed{\|F(x) - F(y)\| \ge \frac{1}{2}\|y-x\|}$$

$$\left(D^1 \tilde{F}^{-1}\right)^{-1} = \begin{pmatrix} E_m & 0 \\ D^1_x F & D^1_y F \end{pmatrix}^{-1} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$\begin{pmatrix} E_m & 0 \\ D^1_x F & D^1_y F \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} E_m & 0 \\ 0 & E_n \end{pmatrix}$$

$A = E_m$, $B = 0$

$D^1_x F + D^1_y F \cdot C = 0$

$D^1_y F \cdot D = E_n$

$$\boxed{C = -\left(D^1_y F\right)^{-1} \cdot \left(D^1_x F\right)}$$

$D = \left(D^1_y F\right)^{-1}$

8.40 $F:\mathbb{R}^2\to\mathbb{R}$, $F(x,y)=xy\sin\left(\frac{\pi}{2}xy^2\right)$

$F(x,y)=1$ ונק' $(a,b)=(1,1)$

$y=f(x)$? $y'=f'(1)$?

$F_x = y\sin\left(\frac{\pi}{2}xy^2\right) + xy\cos\left(\frac{\pi}{2}xy^2\right)\frac{\pi}{2}y^2 \Big|_{(1,1)} = 1$

V C^1, $(1,1)$ [illegible]

$F_y(1,1) = x\sin\left(\frac{\pi}{2}xy^2\right)\Big|_{(1,1)} + x^2y^2\cos\left(\frac{\pi}{2}xy^2\right)\cdot\pi\,\cancel{xy}\Big|_{(1,1)}$

$= 1$ $\Rightarrow f'(1) = -\frac{1}{1} = -1$ ✓

$$8.42 \quad F(x,y,z) = \sin(xy) + \sin(yz) + \sin(xz)$$

$$F(x,y,z) = 0 \rightsquigarrow z(x,y) \; ? \; \sim \mathcal{M}' \; \underline{\underline{[\pi,1,0]}}$$

$$F_z = \cos(yz)\,y + \cos(xz)\,x \Big|_{[\pi,1,0]} = 1+\pi \neq 0$$

$$z_x = -\frac{F_x}{F_z} = -\frac{1}{1+\pi}, \quad z_y = -\frac{F_y}{F_z} = \frac{\pi}{1+\pi}$$