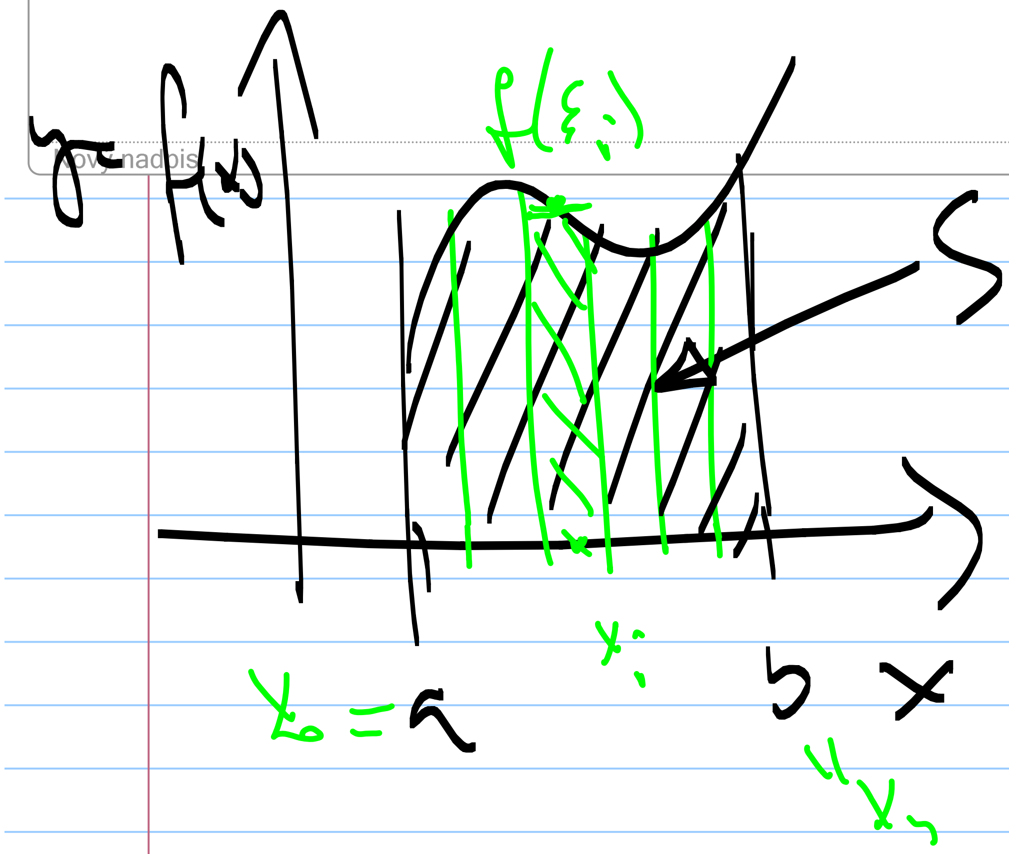
Nový nadpis 8.10.2013

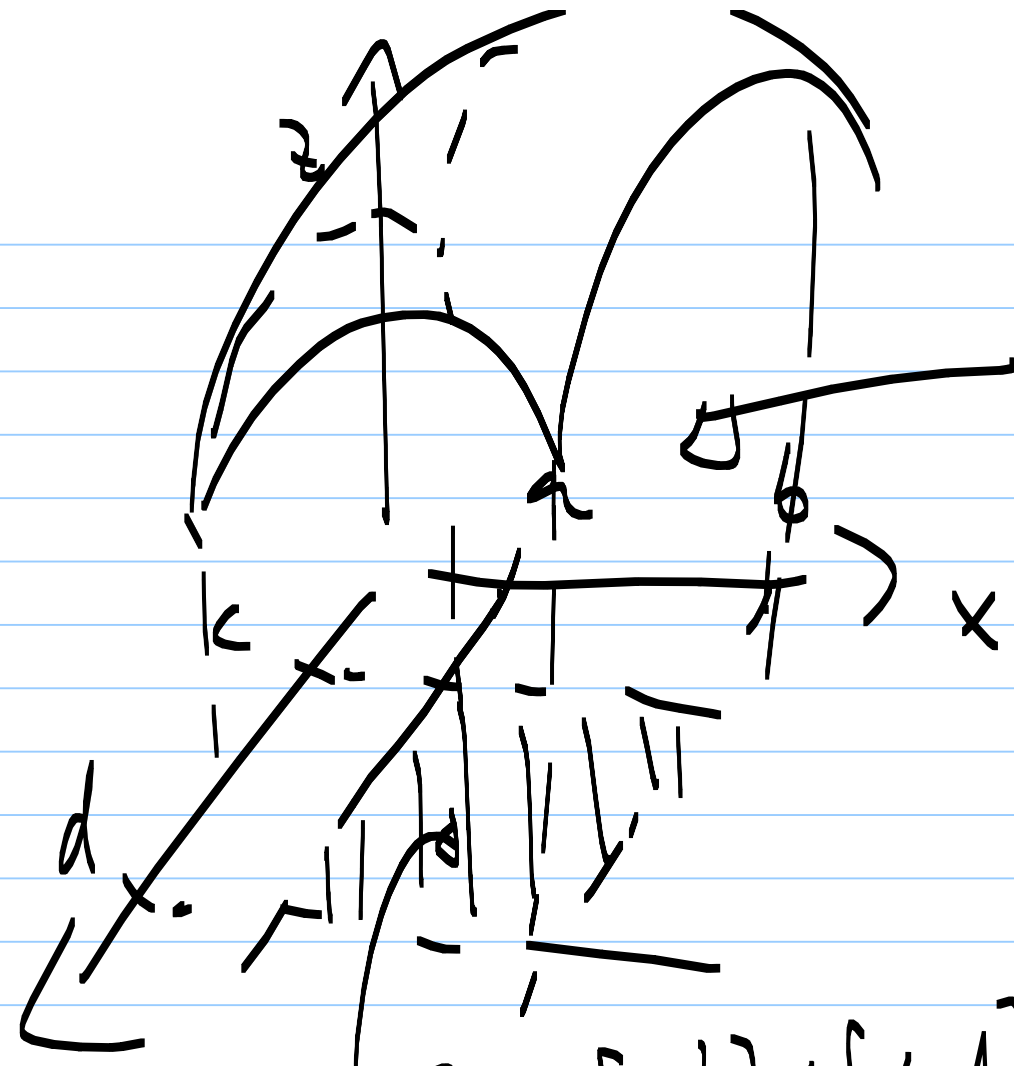


$$F'(x) = f(x)$$

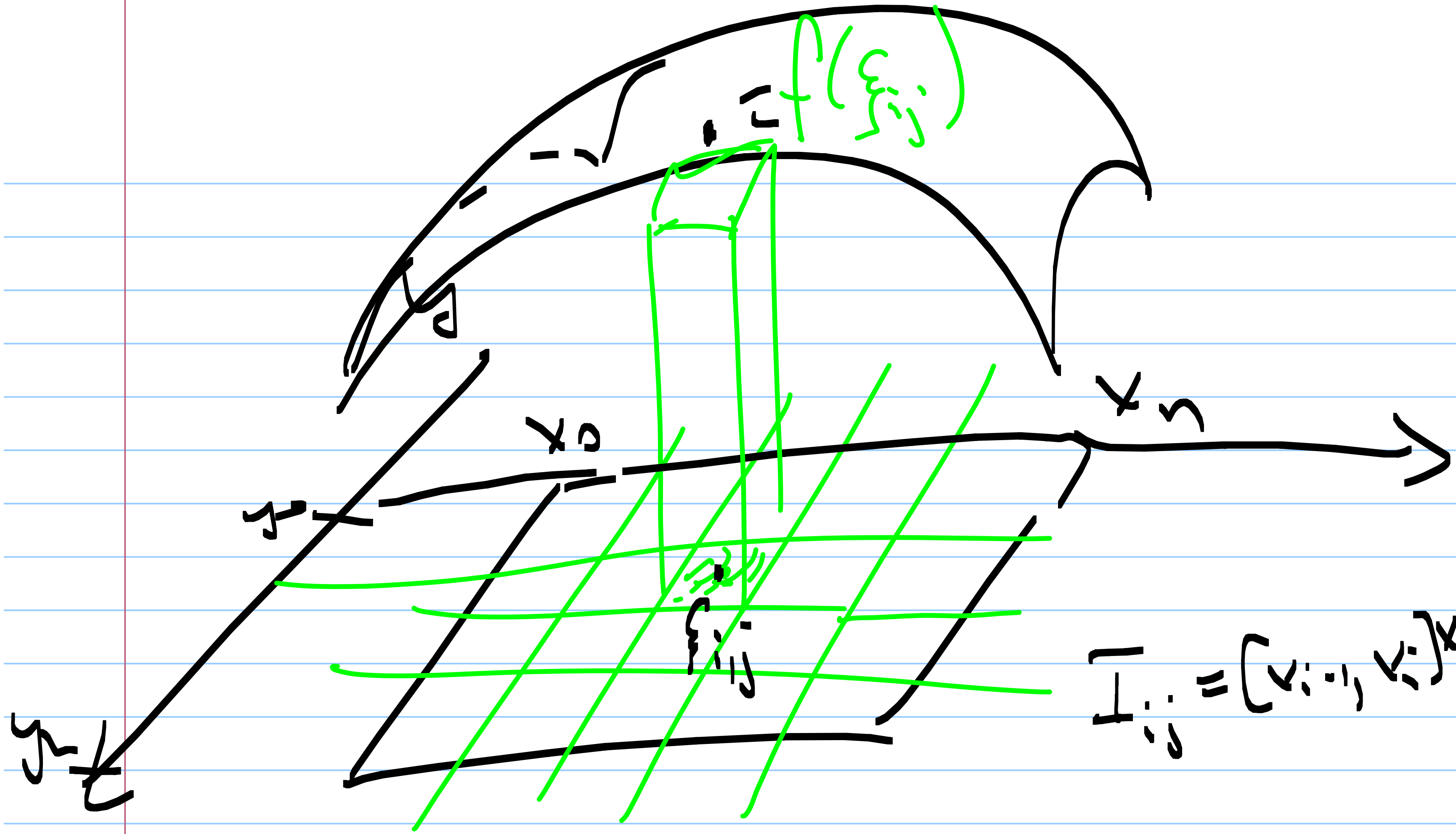
$$S = \int_a^b f(x)\,dx = F(b) - F(a)$$

$z = f(x,y)$

$$\boxed{V = \int_M f(x,y)\,dx\,dy}$$



$M = [a,b] \times [c,d]$

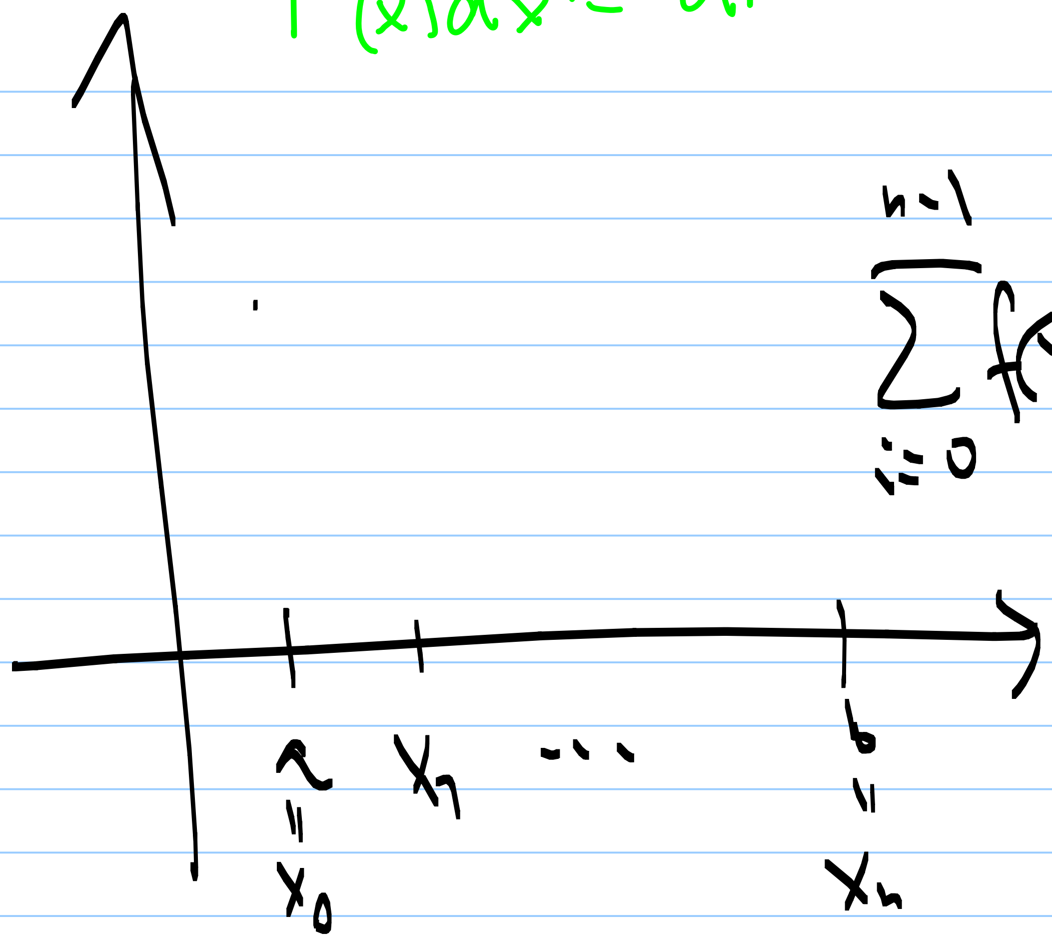


$$I_{ij} = [x_{i-1}, x_i] \times [y_{j-1}, y_j]$$

$$\left| \sum_{i=1}^{n} f(\xi_i, \bar{y})(x_i - x_{i-1}) - \sum_{i=1}^{n} f(\xi_i, y)(x_i - x_{i-1}) \right|$$

$$\leq \sum_{i=1}^{n} \underbrace{\left| f(\xi_i, \bar{y}) - f(\xi_i, y) \right|}_{\leq \varepsilon} (x_i - x_{i-1})$$

$$\leq \varepsilon (b - a) \qquad \square$$



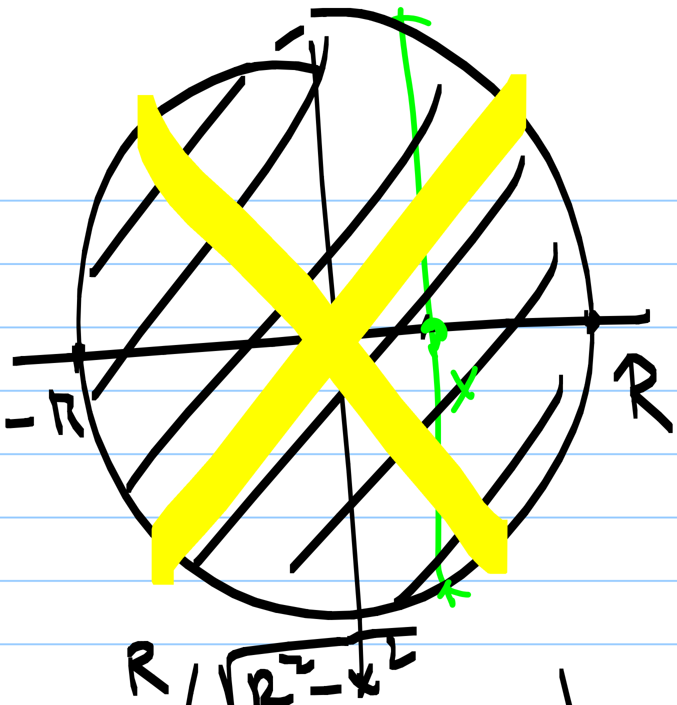
$$F'(x)\,dx = dF$$

$$f(x_i) \cong \frac{\bar{F}(x_{i+1}) - \bar{F}(x_i)}{x_{i+1} - x_i}$$

$$\sum_{i=0}^{n-1} f(x_i)(x_{i+1} - x_i) =$$

$$= \sum_{i=0}^{n-1} \frac{\bar{F}(x_{i+1}) - \bar{F}(x_i)}{\cancel{x_{i+1} - x_i}}$$

$$= \bar{F}(x_n) - \bar{F}(x_0)$$



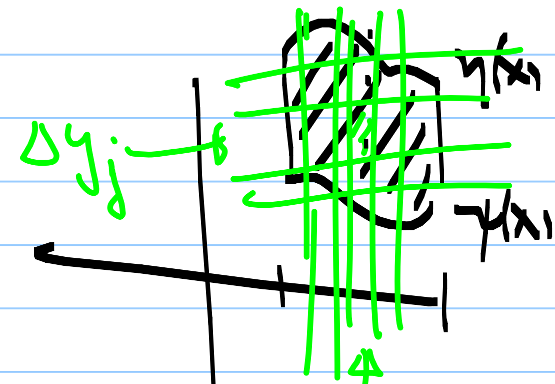
$$M = \{ x^2 + y^2 \le R^2 \}$$

$$\iint_M xy \, dx \, dy$$

$$\int_{-R}^{R} \left(\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} xy \, dy \right) dx = \int_{-R}^{R} x \cdot 0 \, dx = 0$$

$$\left[\frac{1}{2} y^2 \right]_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-y^2}}$$

$$I = [a,b] \times [c,d] \supset M = \{(x,y) \in I;\ y \in [\psi(x), \eta(x)]\}$$

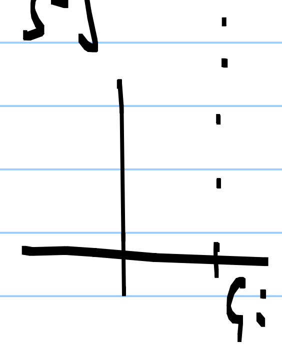


$$S_{\Xi,\xi} = \sum_{i,j} f(\xi_{ij})\, \Delta x_{ij}$$

$$= \sum_i \underbrace{\left(\sum_j f(\xi_{ij})\, \Delta y_j\right)}_{\sim \int_{\psi(\xi_i)}^{\eta(\xi_i)} f(\xi_i, y)\, dy \;=\; S_i} \Delta x_i$$

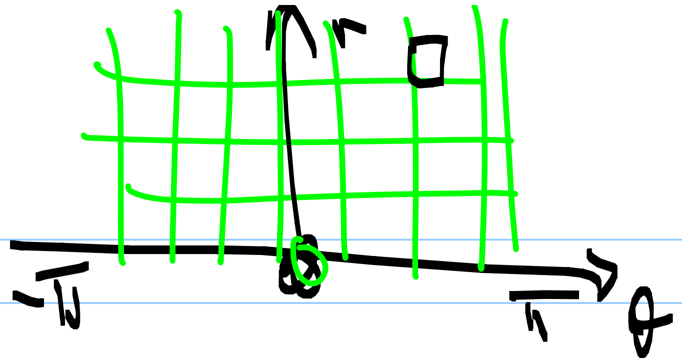
$\boxed{\Delta x_{ij} = \Delta x_i\, \Delta y_j}$

ξ_{ij} "med" ξ_i



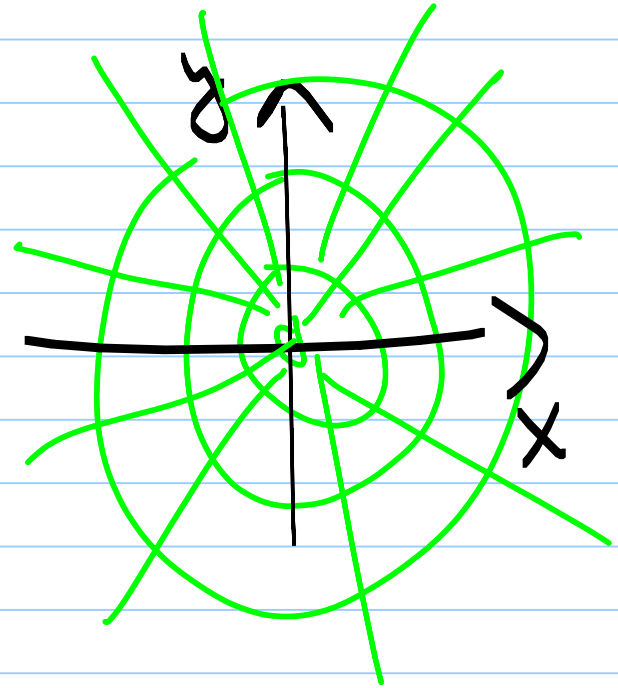
$$\sum_i S_i\, \Delta x_i \longrightarrow \int_a^b \left(\int_{\psi(x)}^{\eta(x)} f(x,y)\, dy \right) dx$$

$$F\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

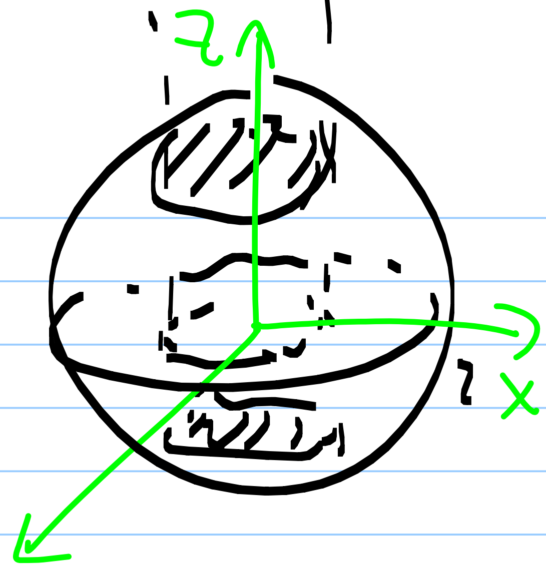


(r, θ)

$|D F|$



$\boxed{8.74.}$ $M = \begin{cases} x^2+y^2+z^2 \le 4 \\ x^2+y^2 \le 1 \end{cases}$



1) $8 \times$ "1. Oktant" $\tilde{M}$

$G \begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$ $\qquad \tilde{M} = \begin{cases} \theta \in [0, \pi/2] \\ r \in [0, 1] \\ 0 \le z \le \sqrt{4-r^2} \end{cases}$ $\qquad D'G = \begin{pmatrix} \cdot & \cdot & 0 \\ \cdot & \cdot & 0 \\ 0 & 0 & 1 \end{pmatrix}$

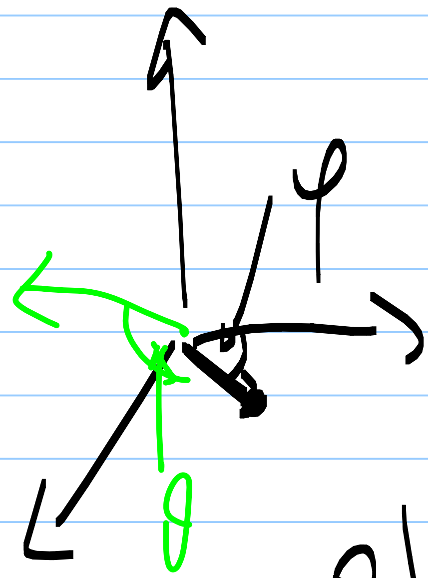
$|D'G| = r$

$$V = 8 \int_0^{\pi/2} \int_0^1 \int_0^{\sqrt{4-r^2}} r \, dz \, dr \, d\theta = 8 \int_0^{\pi/2} \int_0^1 r\sqrt{4-r^2} \, dr \, d\theta$$

$= \frac{2}{3}(8-3\sqrt{3})\pi$

$\boxed{8.78.}$ $M: \boxed{x^2+2y^2+3z^2 \leq 1}$

$$G\begin{cases} x = r\cos\varphi\sin\theta \\ y = \frac{1}{\sqrt{2}} r \sin\varphi \sin\theta \\ z = \frac{1}{\sqrt{3}} r\cos\theta \end{cases}$$



$$D^1G = \begin{pmatrix} \cos\varphi\sin\theta & -r\sin\varphi\sin\theta & +r\cos\varphi\cos\theta \\ \vdots & & \vdots \end{pmatrix}$$

$$|D^1G| = \frac{1}{\sqrt{6}} r^2 \sin\theta \qquad V = \int_0^{2\pi}\int_0^{\pi}\int_0^{1} \frac{1}{\sqrt{6}} r^2 \underline{\underline{\sin\theta}}\, dr\, d\theta\, d\varphi$$