Nový nadpis 9.10.2013

Fubini: $\int_a^b\left(\int_c^d f(x,y)\,dy\right)dx = \int_c^d\left(\int_a^b f(x,y)\,dx\right)dy$

$=$

$F(y) = \int_a^b f(x,y)\,dx$ $\boxed{\frac{\partial F}{\partial y} \; ?}$ $G(y) = \int_a^b \frac{\partial f}{\partial y}(x,y)\,dx$

$H(y) = \int_{y_0}^{y} G(s)\,ds = \int_{y_0}^{y}\left(\int_a^b \frac{\partial f}{\partial y}(x,s)\,dx\right)ds$

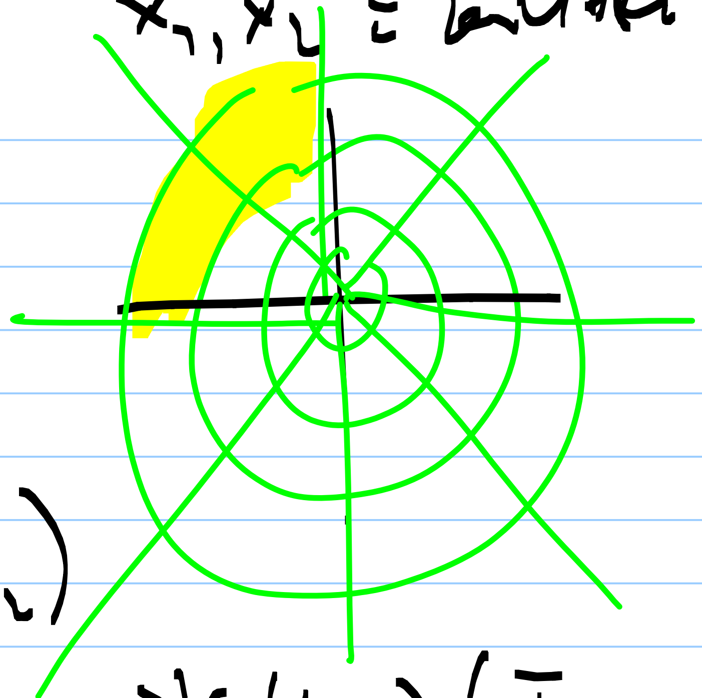
$= \int_a^b\left(\int_{y_0}^{y}\frac{\partial f}{\partial y}(x,s)\,ds\right)dx = \int_a^b\left(f(x,y)-f(x,y_0)\right)dx$

$= F(y) - F(y_0)$

t_1, t_2 = polární v $\mathbb{R}^2$

$\xrightarrow{G}$

x_1, x_2 = kartézské v $\mathbb{R}^2$



$$I_{i_1 i_2} \overset{G}{\longmapsto} J_{i_1 i_2} = G(I_{i_1 i_2})$$

$$\downarrow$$

$$t_{i_1 i_2} \qquad I_{i_1 i_2} \longmapsto G(t_{i_1 i_2}) + D^1G(t_{i_1 i_2})(I_{i_1 i_2} - t_{i_1 i_2})$$

$$= R_{i_1 i_2}$$

$$\boxed{\operatorname{vol} R_{i_1 i_2} = |\det D^1 G| \operatorname{vol} I_{i_1 i_2}}$$

pro $\varepsilon > 0$ ex. $\delta > 0$, že pro dělení s normou $< \delta$ platí

$$G_{i_1 i_2}(t_{i_1 i_2}) + (1+\varepsilon) D^1 G(I_{i_1 i_2}) \supset J_{i_1 i_2}$$

$$\operatorname{vol}(J_{i_1 i_2}) \le (1+\varepsilon)^n \operatorname{vol} R_{i_1 i_2} = (1+\varepsilon)^n |\det D^1 G| \operatorname{vol} I_{i_1 i_2}$$

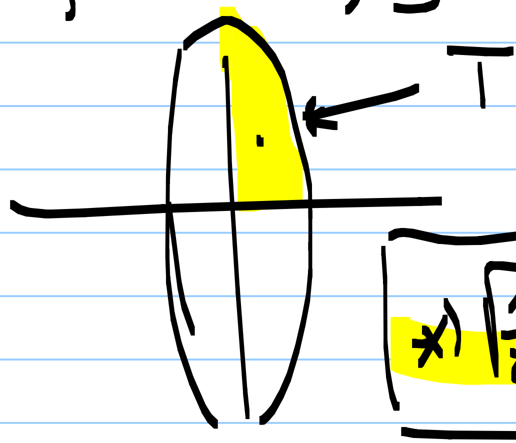
$$\boxed{\int_M f \, dx_1 \, dx_2} = \sum_{i_1 i_2} \int_{I_{i_1 i_2}} f \, dx_1 \, dx_2 \le \sum_{i_1 i_2} \overbrace{\sup f}^{\le f \circ G} \cdot \operatorname{vol} J_{i_1 i_2}$$

$$\le (1+\varepsilon) \sum_{i_1 i_2} \sup f \, |\det D^1 G| \operatorname{vol} I_{i_1 i_2} \xrightarrow[\delta \to 0]{\varepsilon \to 0}$$

$$= 1 \quad M = G(N) \quad \int_N (f \circ G) |\det D^1 G| \, dt_1 \, dt_2$$

8.85. elipsa $3x^2+2y^2=1$, $x\geq 0$, $y\geq 0$.

Těžiště. $T=[T_x,T_y]$



$*)\ \sqrt{\frac{1}{2}(1-3x^2)}$

$T_x=\frac{1}{V}\iint_M x\,dx\,dy \qquad T_y=\frac{1}{V}\iint_M y\,dx\,dy$

$V=\iint_M dx\,dy$

$x=\frac{1}{\sqrt{3}}\bar{x} \qquad y=\frac{1}{\sqrt{2}}\bar{y} \qquad \begin{vmatrix}\frac{1}{\sqrt{3}} & 0\\ 0 & \frac{1}{\sqrt{2}}\end{vmatrix}=\frac{1}{\sqrt{6}}$

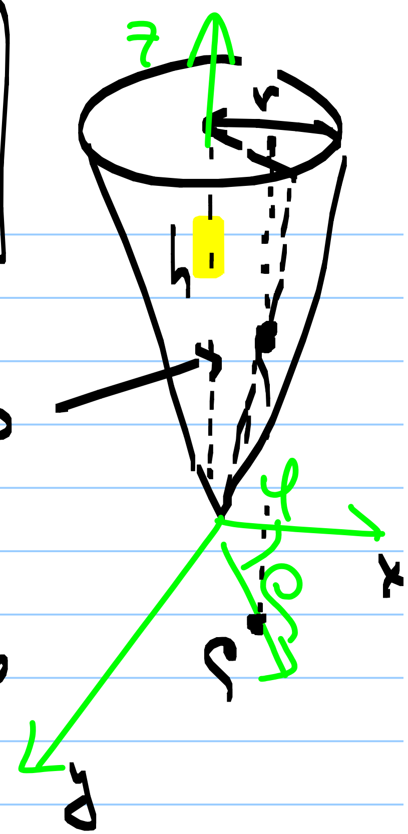
$\bar{M}=\{\bar{x}^2+\bar{y}^2=1,\ \bar{x}\geq 0,\ \bar{y}\geq 0\}$

$\iint_M dx\,dy=\iint_{\bar{M}}\frac{1}{\sqrt{6}}\,d\bar{x}\,d\bar{y}=\frac{1}{\sqrt{6}}\cdot\frac{1}{4}\pi$

$T_x=\int_0^{1/\sqrt{3}}\int_0^{*)} x\,dy\,dx=\int_0^{1/\sqrt{3}}\frac{x}{\sqrt{2}}\sqrt{1-3x^2}\,dx=\boxed{\frac{\sqrt{2}}{18}}$

$\boxed{T_y=\frac{\sqrt{3}}{18}}$

8.86. $x = \rho\cos\varphi$, $y = \rho\sin\varphi$, $z = z$ $\quad |D| = \rho \quad \boxed{\frac{h}{r}\rho \le z \le h}$



$$V = 4\int_0^{\pi/2}\int_0^{r}\int_{\rho h/r}^{h} \rho\, dz\, d\rho\, d\varphi =$$

$$= 4h\int_0^{\pi/2}\int_0^{r} \rho\left(1 - \rho\frac{1}{r}\right) d\rho\, d\varphi = 4h\int_0^{\pi/2}\left(\frac{1}{2}r^2 - \frac{1}{3}r^2\right) d\varphi$$

$$= \frac{2}{3}hr^2\frac{\pi}{2} = \frac{1}{3}\pi h r^2$$

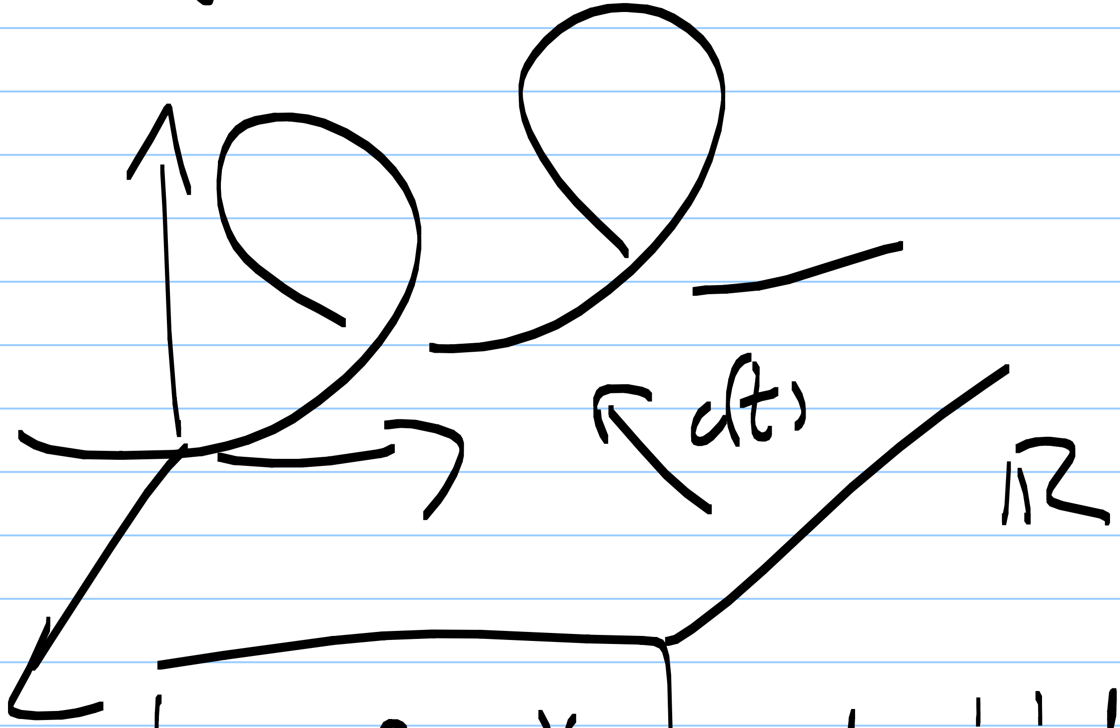
$$\bar z = \frac{1}{V}\int_M z\, dx\, dy\, dz = \frac{1}{V}\int_0^{\pi/2}\int_0^{r}\int_{\rho\frac{h}{r}}^{h} z\rho\, dz\, d\rho\, d\varphi = \frac{h^2}{2V}\int_0^{\pi/2}\int_0^{r} \rho\left(\cancel{h^2} - \rho^2\frac{\cancel{h^2}}{r^2}\right) d\rho\, d\varphi$$

$$= \frac{h^2 4}{2V} \int_0^{\pi/2} \int_0^{r} \left(\rho - \rho^3 \frac{1}{r^2}\right) d\rho \, d\varphi = 4 \cdot \frac{3}{2} \frac{h}{\pi r^2} \frac{\pi}{2} \left(\left[\frac{1}{2}\rho^2\right]_0^r - \left[\frac{1}{4}\rho^4\right]_0^r \frac{1}{r^2} \right)$$

$$= \frac{3 \cdot \cancel{4}\, h}{4\, \cancel{r^2}} \frac{1}{\cancel{4}} \cancel{r^2} = \underline{\underline{\frac{3}{4} h}}$$

$\boxed{8.88.}$ $M: \left\{ z \geq 0 \quad z \leq 2 \quad \begin{matrix} x^2+y^2 \leq z^2 \\ x^2+y^2 \geq 2z^2 \end{matrix} \right\}$

Integrály přes $M \subset \mathbb{R}^n$ vyšší dim.



$\int_c f \, vlc$

$d(f \circ c) = \left(\frac{\partial f}{\partial x} c_1'(t) + \frac{\partial f}{\partial y} c_2'(t) + \dots\right) dt$

$\boxed{\operatorname{grad} f \cdot c'(t)}$

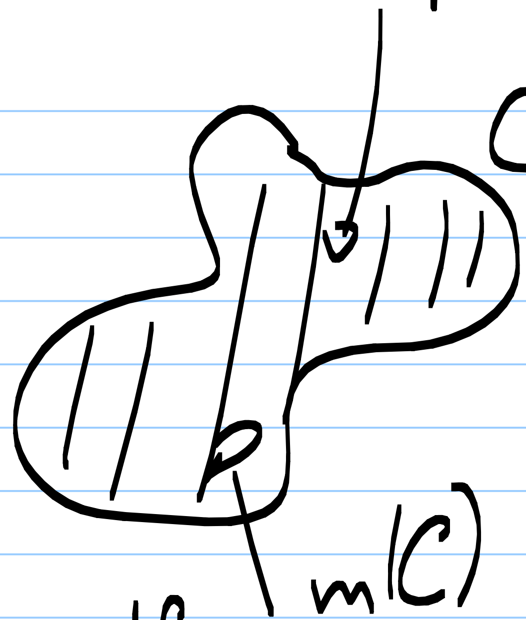
lineární fce ve $\mathbb{R}^3$:

$\omega = \boxed{f(x,y,z)\,dx + g(x,y,z)\,dy + h(x,y,z)\,dz}$

$\forall c: \int_c \omega$

C ... uzavřená křivka v $\mathbb{R}^2$

$$\boxed{m(C) = \int_C -y\,dx + x\,dy}$$



$$\omega = -y\,dx + x\,dy \in \Omega^1(\mathbb{R}^2)$$

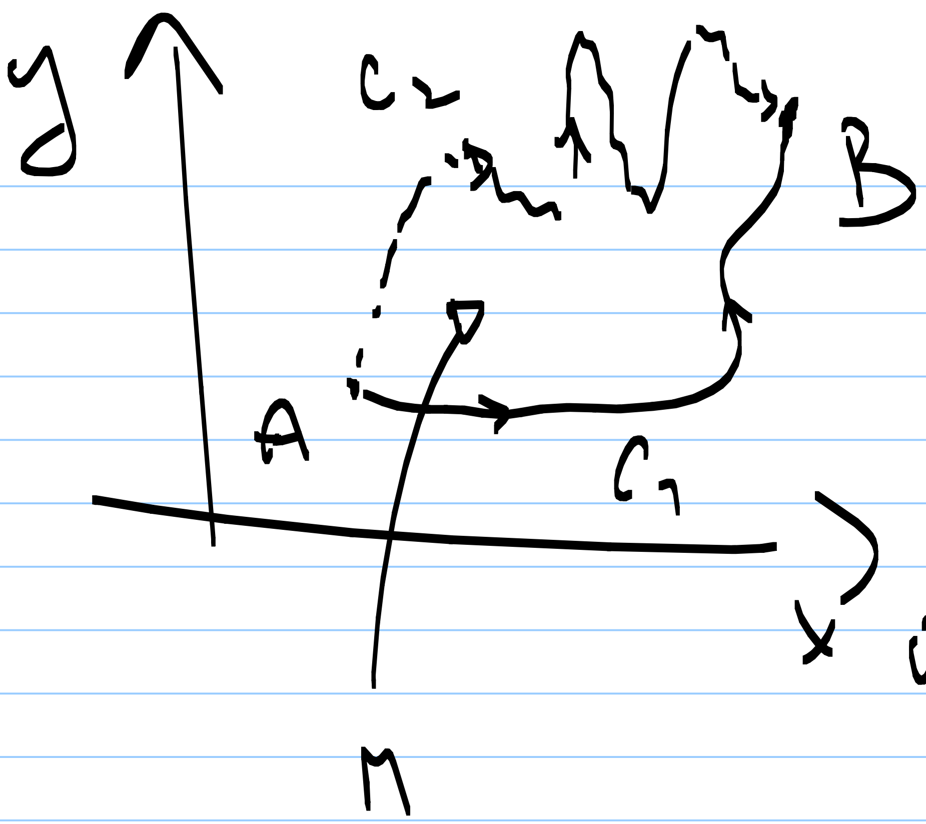
$$\omega = f(x,y)\,dx + g(x,y)\,dy$$

$d : f \mapsto df = f_x\,dx + f_y\,dy$

$$d\omega = \frac{\partial f}{\partial y}\,dy \wedge dx + \frac{\partial g}{\partial x}\,dx \wedge dy$$

Greenova V.

$$\boxed{\int_{\partial M = C} \omega = \int_M d\omega}$$



$F : \mathbb{R}^2 \to \mathbb{R}$

dF

$C : C_1 \vee -C_2$

$A \to B \to A$

$$0 = \int_{C_1} dF - \int_{C_2} dF = \int_C dF = \int_M 0$$

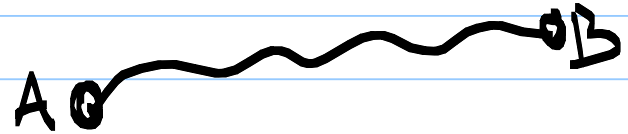
k-dim M

$$\Omega^k(\mathbb{R}^n) \xrightarrow{d} \Omega^{k+1}(\mathbb{R}^n)$$

dim $k+1$ M

dim k ∂M

$$\int_{\partial M} \omega = \int_M d\omega$$



$$\int_C dF = \int_{\partial C} F = F(B) - F(A)$$