$\boxed{8.53}$ $3x^2 + 2xy + x - y^2 - 3y - \frac{5}{4} = F(x,y)$

$y = f(x)$? extrémy f (lokální)

$\boxed{y'} = f'(x) = -\frac{F_x(x,y)}{F_y(x,y)} = \boxed{-\frac{6x+2y+1}{-2y+2x-3}} = 0$

$y = -3x - \frac{1}{2}$ dosadíme do $F(x,y) = 0$

$3x^2 - 2x\left(3x+\frac{1}{2}\right) + x - 9x^2 - 3x - \frac{1}{4} + 9x + \frac{3}{2} - \frac{5}{4} =$

$= -12x^2 + 6x = 0 \Rightarrow$ $\boxed{\begin{matrix} x = 0, & x = \frac{1}{2} \\ y = -\frac{1}{2}, & y = -2 \end{matrix}}$

$$y'' = (y')' = \left(\frac{6x+2y+7}{2y-2x+3}\right)' = \frac{(6+2y')(2y-2x+3) - (6x+2y+7)(2y'-2)}{(2y-2x+3)^2}$$

$\rightsquigarrow$ להציב את y'

את $[x,y] = [0,-\frac{1}{2}]$, $[\frac{1}{2},-2]$

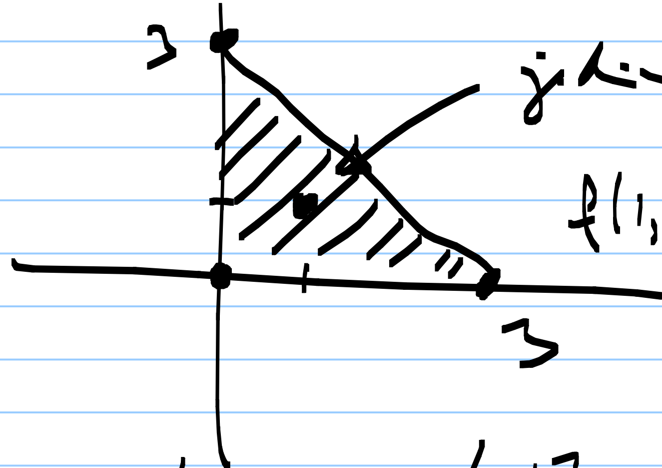
$\boxed{8.49}$ $f(x,y) = x^2 - 2y^2 + 4xy - 6x - 1$

$D = \{(x,y) : x \geq 0,\ y \geq 0,\ y \leq -x + 3\}$

$[1,1]$ -4, $[0,0]$ -1, $[0,3]$ -19,
$(3,0)$ -10, $\left[\frac{9}{5}, \frac{6}{5}\right]$ $-14/5$

$f_x = -2x + 4y - 6 = 0 \qquad x = 1$

$f_y = -4y + 4x = 0 \qquad y = x$



[illegible]

$f(1,1) = -4$

$g_1(t) = f(t,0) = t^2 - 6t - 1$

$g_2(t) = f(0,t) = -2t^2 - 1$

$x = t,\ y = -t + 3$

$g_3(t) = t^2 - 2(-t+3)^2 + 4t(-t+3) - 6t - 1$

8.18.) $g(x,y,z) = kx^2 + ly^2 + z^2 = 1 \qquad k, l \in \mathbb{R}_+$

$\boxed{f(x,y,z) = xyz}$ extrémy f na $M = \{g = 1\}$

vektor (g_x, g_y, g_z) je kolmý k tečné rovině M

vektor (f_x, f_y, f_z) je kolmý k M

$\Leftrightarrow$ $f|_M$ má stac. bod

$g = kx^2 + ly^2 + z^2 - 1$

$f = xyz$

$$\frac{\partial g}{\partial x} = \lambda \frac{\partial f}{\partial x} : \quad 2kx = \lambda yz$$

$$\frac{\partial g}{\partial y} = \lambda \frac{\partial f}{\partial y} : \quad 2ly = \lambda xz$$

$$\frac{\partial g}{\partial z} = \lambda \frac{\partial f}{\partial y} : \quad 2z = \lambda xy$$

$+ \quad \boxed{g = 0}$

$$2k \frac{x}{y} = 2l \frac{y}{x} \rightsquigarrow \boxed{x = \pm \frac{1}{\sqrt{3k}}, \ y = \pm \frac{1}{\sqrt{3l}}, \ z = \pm \frac{1}{\sqrt{3}}}$$

$$\Rightarrow \pm \frac{1}{3\sqrt{3kl}}$$