MA010 Tutorial 6

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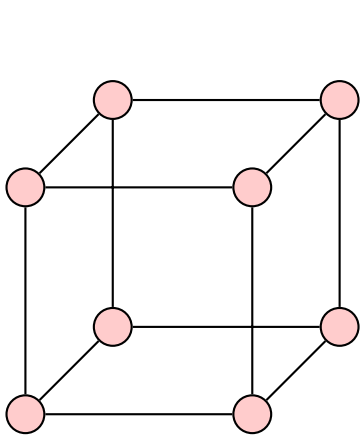
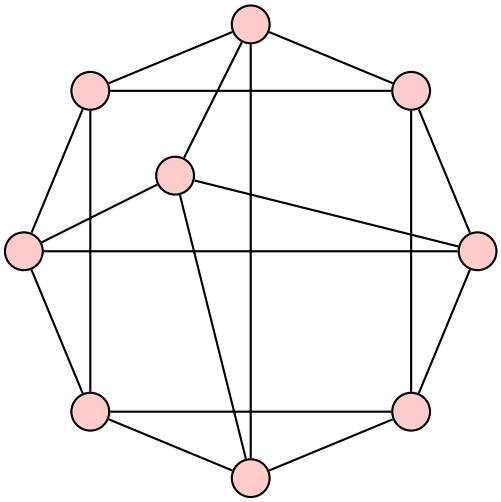
This tutorial covers material from lecture 7 (planarity).

Problem 1

Are these graphs planar or not? If they are planar, give a planar drawing, if not, prove that they are not (possibly by using Kuratowski's theorem...)



(a)

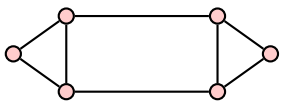


(b)

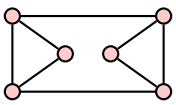
Sources: www.math.uwaterloo.ca/~dgwagner/CO220/co220sols6.pdf, www.math.hawaii.edu/~marriott/teaching/summer2010/math100/planar_graphs_homework.pdf

Problem 2

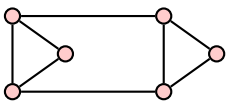
Of the following three isomorphic planar graphs, which ones are equivalent drawings and which are not? Why?



(a)



(b)



(c)

Problem 3

Show that a connected planar bipartite graph with $n \geqslant 3$ vertices can have at most $e \leqslant 2n - 4$ edges. Show that this is the best possible bound by giving a bipartite planar graph where this equality is attained.

Source: www3.nd.edu/~dgalvin1/40210/40210_S12/40210S12-E1_sols.pdf

Problem 4

Call a graph "outer planar" if it can be drawn on the plane with no crossings such that all vertices are on the outer face. Show that every outer planar graph is 3-colourable.

Source: wetalldid.com/study/maryland/jhu/math_550.472_graph_theory/amitabh_basu/math_550.472_graph_theory_homework_10_amitabh_basu_sp2014.pdf

Problem 5

Show that a simple, 2-connected, 6-regular planar graph cannot exist. (Recall that 6-regular means that every vertex has degree 6.)

Hint: How many faces can there be compared to the number of edges? Now try counting the number of faces a second way...