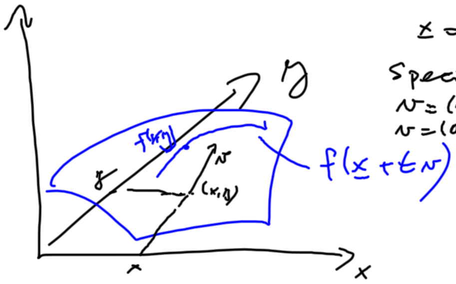
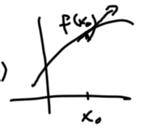
$$f(x,y) = f(\underline{x})$$
$$\underline{x} = (x,y)$$

Speciálně:
$v = (1,0)$ } parc.
$v = (0,1)$ } derivace



$$h: \mathbb{R} \to \mathbb{R}$$
$$h'(x_0) = \lim_{r \to 0} \frac{h(x+r) - h(x)}{r}$$



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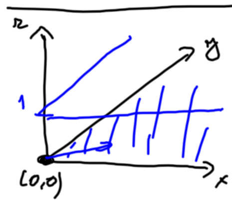
$$d_v f(\underline{x}) = \lim_{t \to 0} \frac{1}{t}\left(f(\underline{x} + tv) - f(\underline{x})\right) \qquad \underline{x} = (x,y)$$

$v = (1,0)$: $d_{(1,0)} f(\underline{x}) = \frac{\partial f(\underline{x})}{\partial x} =$
$$= \lim_{t \to 0+} \frac{1}{t}\left(f(x+t,y) - f(x,y)\right)$$



$r = g(x,y)$

$x \cdot y = 0 \Leftrightarrow x = 0 \lor y = 0$

$\frac{\partial g}{\partial x}(0,0) = 0$

$v = (1,1)$

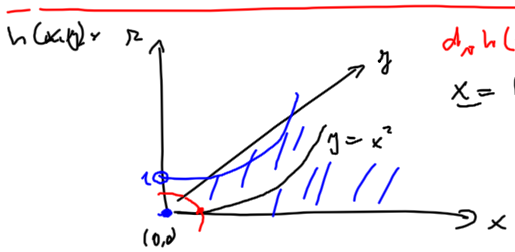
$\frac{\partial g}{\partial y}(0,0) = 0$

9 29-14:18

$v = (1,1)$ $\qquad \underline{x} = (0,0)$

$$d_v g(0,0) = \lim_{t \to 0} \frac{1}{t}\left(f(t,t) - f(0,0)\right)$$
$$= \lim_{t \to 0} \frac{1}{t}(0 - 1) \text{ neexistuje}$$
(resp. nevlastní limita)

$h(x,y)$:



$d_v h(0,0)$

$\underline{x} = (0,0)$

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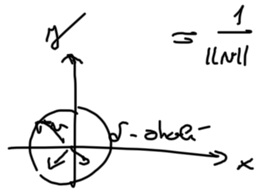
Diferenciál: (4)

$$\lim_{v \to 0} \frac{1}{\|v\|}\left(f(\underline{x} + v) - f(\underline{x})\right) =$$
$$= \frac{1}{\|v\|} \cdot d_v f(\underline{x})$$



$\forall \varepsilon > 0 \; \exists \delta > 0$ t.ž. ...

9 29-14:37

(5)

$v = (v_1, v_2)$ $\qquad f(x,y)$

$$d_v f(x_0,y_0) = \frac{\partial f(x_0,y_0)}{\partial x} \cdot v_1 + \frac{\partial f(x_0,y_0)}{\partial y} \cdot v_2$$

Věta: "parc. derivace ⇝ diferenciál"

$h(x,y) \to$ má všechny směrové derivace v bodě $(0,0)$, které jsou nulové

Ale směrové derivace neexistují na okolí b. $(0,0)$.

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(6)

$f: E_2 \to \mathbb{R}$

$$r(x,y) = \overbrace{f(x_0,y_0)}^{\text{konst.}} + \frac{\partial f}{\partial x}(x_0,y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y - y_0)$$

- $r(x_0,y_0) = f(x_0,y_0)$

$\frac{\partial r}{\partial x}(x_0,y_0) = \frac{\partial f}{\partial x}(x_0,y_0)$

9 29-14:54

$d_v f = d_v f(\underline{x})$, $\underline{x} \in E^n$ (7)

první rozlišný bod

$f: E_n \to \mathbb{R}$

$d_v f: E_n \to \mathbb{R}$

$\rightarrow$ má smysl $d_v(d_v f)$

· $v = (1, 0, \dots, 0)$; $\frac{\partial}{\partial x_1}\left(\frac{\partial}{\partial x_1} f\right) = \frac{\partial^2}{\partial x_1^2} f$

· $d_w(d_v f)$; $v = (\dots, \underset{i}{1}, \dots)$

$w = (\dots, \underset{j}{1}, \dots)$

$\frac{\partial}{\partial x_j}\left(\frac{\partial}{\partial x_i} f\right) = \frac{\partial^2 f}{\partial x_j \partial x_i}$

9 29-15:05

· Taylorův rozvoj $\rightsquigarrow$ aproximace zadané funkce polynomem

· $f: \mathbb{R} \to \mathbb{R}$ jedné proměnné

$x_0 \in \mathbb{R}$

$T f(x_0 + t) = f(x_0) + t f'(x_0) + \frac{1}{2} t^2 f''(x_0) + \frac{1}{3!} t^3 f'''(x_0) + \dots$

$v = (\xi, \eta)$, $f: E_2 \to \mathbb{R}$

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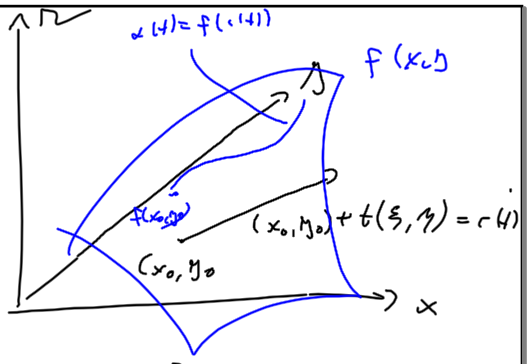

$f_{xx} = \frac{\partial^2 f}{\partial x^2}$

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$D^1 f(x)(v) = d_v f(x)$ (10)

$D^2 f(x)(v) = d_v \underbrace{(d_v f)}_{E_2 \to \mathbb{R}} (x)$

$v = (\xi, \eta)$, $\underbrace{d_v f(x)}_{D^1 f(x)(v)} = \underbrace{\frac{\partial}{\partial x} f \cdot \xi + \frac{\partial}{\partial y} f \cdot \eta}_{d_v(\dots)}$

$D^2 f(x)(v) = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} f \cdot \xi + \frac{\partial}{\partial y} f \cdot \eta\right) \cdot \xi + \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x} f \cdot \xi + \frac{\partial}{\partial y} f \cdot \eta\right) \cdot \eta$

$= \frac{\partial^2 f}{\partial x^2} \cdot \xi^2 + 2 \frac{\partial^2 f}{\partial x \partial y} \cdot \xi \eta + \frac{\partial^2 f}{\partial y^2} \cdot \eta^2$

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