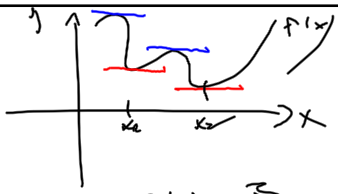
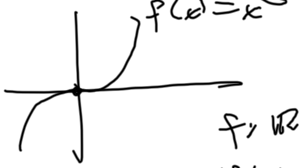


$f'(x_1) = f'(x_2) = 0$



$f(x) = x^3$

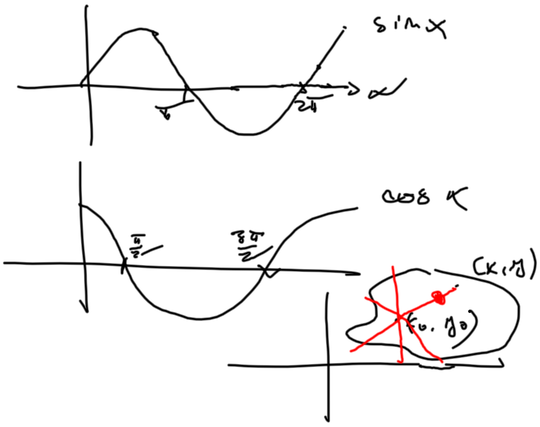
$f'(0) = 0$

$f''(0) = 0$

$f: \mathbb{R}^n \to \mathbb{R}$, $x_0 \in \mathbb{R}^n$

$df(x_0): \mathbb{R}^n \to \mathbb{R}$

10 6-14:06



10 6-14:15

$df(x_0,y_0): \mathbb{R}^2 \to \mathbb{R}$ lineární forma

$Hf(x_0,y_0): \mathbb{R}^2 \to \mathbb{R}$ kvadratická forma

$Hf(x_0,y_0)(v) =$

$= v^T \cdot Hf(x_0,y_0) \cdot v$

10 6-14:25

$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = A$ poz. def. jestliže

$|a| > 0$

$\begin{vmatrix} a & b \\ d & e \end{vmatrix} > 0$

$|A| > 0$

negativní definitnost

znaménka se střídají!

10 6-14:32

(x,y) — kartézské

(r,φ) — polární

$x = r\cos\varphi$
$y = r\sin\varphi$

$r = \sqrt{x^2+y^2}$

$\frac{y}{x} = \frac{\sin\varphi}{\cos\varphi} = \operatorname{tg}\varphi$

$\varphi = \operatorname{arctg}\left(\frac{y}{x}\right)$

10 6-14:41

$D^1F(x): \mathbb{R}^n \to \mathbb{R}^m$

m / n

$\lim_{v\to 0} \frac{1}{\|v\|}\left(F(x+v) - F(x)\right) =$

$= \lim_{v\to 0} \frac{1}{\|v\|} \underbrace{D^1F(x_0)}_{\text{lin. zobr.}}(v)$

$= D^1F(x_0)\left(\lim_{v\to 0} \frac{v}{\|v\|}\right)$

$\mathbb{R}^n$

$\frac{v}{\|v\|}$ konst

10 6-14:46

$F, G : \mathbb{R} \to \mathbb{R}$

$(G \circ F)'(x) = G'(F(x)) \cdot F'(x)$

Obecně

$D^1F(x) : \mathbb{R}^n \to \mathbb{R}^m$

$D^1G(F(x)) : \mathbb{R}^m \to \mathbb{R}^r$

lim, zobr.

10 6-14:53

Důkaz: $n = r = 1$, $m = 2$ $\quad (x_0, y_0) = (x(0), y(0))$

$G : \mathbb{R}^2 \to \mathbb{R}$, $G(x, y)$

$F : \mathbb{R} \to \mathbb{R}^2$, $F(t) = (x(t), y(t))$

$\rightsquigarrow G(x(t), y(t)) = (G \circ F)(t) : \mathbb{R} \to \mathbb{R}$.

$(G \circ F)'(0) = \lim_{t \to 0} \frac{(G \circ F)(t) - (G \circ F)(0)}{t}$

$= \lim_{t \to 0} \underbrace{\frac{G(x(t), y(t)) - G(x_0, y_0)}{t}}$

10 6-14:56

$\frac{1}{t}\left[G(x(t), y(t)) - G(x_0, y_0)\right] =$

$= \frac{1}{t}\left[\left(G(x(t), y(t)) - G(x_0, y(t))\right) +\right.$

$\left. + \left(G(x_0, y(t)) - G(x_0, y_0)\right)\right] =$

$= \frac{1}{t}\left[(x - x_0)\frac{\partial G}{\partial x}(\xi, y(t)) + (y - y_0)\frac{\partial G}{\partial y}(x_0, \eta)\right]$ $\quad \lim_{t \to 0} =$

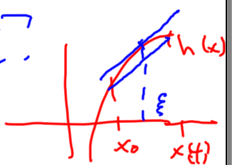
$x_0 \le \xi \le x(t)$

$y_0 \le \eta \le y(t)$

$x(t) \to x_0$, $y(t) \to y_0$ $\quad \xi \to ?$, $\eta \to ?$

$x = x(t)$

$y = y(t)$



$\exists \xi$, $x_0 \le \xi \le x(t)$

t.ž. $f'(\xi) = \frac{h(x) - h(x_0)}{x - x_0}$

$(x - x_0) \cdot f'(\xi) = h(x) - h(x_0)$

Věta o střední hodnotě

10 6-15:01

$= x'(0)\frac{\partial G}{\partial x}(x_0, y_0) + y'(0) \cdot \frac{\partial G}{\partial y}(x_0, y_0)$

$= \left(\frac{\partial G}{\partial x}(x_0, y_0), \frac{\partial G}{\partial y}(x_0, y_0)\right) \cdot \begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix}$

$(x_0, y_0) = F(0)$

10 6-15:10

$t \in \mathbb{R}$

$g(r, \varphi) = \sin(r - t)$ $\quad$ Funkce $\mathbb{R}^2 \to \mathbb{R}$ v polárních souřadnicích

$\frac{\partial g}{\partial r}(r, \varphi) = \cos(r - t)$

$\frac{\partial g}{\partial \varphi}(r, \varphi) = 0$

derivace g v polárních souřadnicích

Složení: $(x, y) \xmapsto{F} (r, \varphi) \xmapsto{G} g(r, \varphi)$

$\mathbb{R}^2 \to \mathbb{R}^2$ $\quad \mathbb{R}^2 \to \mathbb{R}$

$\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y} = ?$ $\quad D^1(G \circ F)(x, y) =$

$= (D^1G)(F(x)) \circ D^1F(x)$

10 6-15:15

$G(r, \varphi) = g(r, \varphi) = \sin(r - t)$

$D^1G(r, \varphi) = \left(\frac{\partial G}{\partial r}(r, \varphi), \frac{\partial G}{\partial \varphi}(r, \varphi)\right) =$

$= (\cos(r - t), 0)$

$F(x, y) = (\underbrace{\sqrt{x^2 + y^2}}_{r}, \underbrace{\operatorname{arctg}\frac{y}{x}}_{\varphi}) : \mathbb{R}^2 \to \mathbb{R}^2$

$DF(x, y) = \begin{pmatrix} \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} \\ \frac{\partial \varphi}{\partial x} & \frac{\partial \varphi}{\partial y} \end{pmatrix} = \begin{pmatrix} \frac{\frac{1}{2} \cdot 2x}{\sqrt{x^2 + y^2}} & \frac{\frac{1}{2} \cdot 2y}{\sqrt{x^2 + y^2}} \\ * & * \end{pmatrix}$

10 6-15:21

$$D^1(G\circ F) = \left(\underbrace{\frac{\partial G}{\partial r},\ \frac{\partial G}{\partial \varphi}\right)\cdot \begin{pmatrix} \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} \\ \frac{\partial \varphi}{\partial x}}_{\frac{\partial (G\circ F)}{\partial x}} & \frac{\partial \varphi}{\partial y} \end{pmatrix} =$$

$$= \left(\frac{\partial G}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial G}{\partial \varphi}\cdot\frac{\partial \varphi}{\partial x},\ \frac{\partial G}{\partial r}\cdot\frac{\partial r}{\partial y} + \frac{\partial G}{\partial \varphi}\cdot\frac{\partial \varphi}{\partial y}\right)$$

$$= \left(\frac{\partial (G\circ F)}{\partial x},\ \frac{\partial (G\circ F)}{\partial y}\right)$$

$$G\circ F : \mathbb{R}^2 \to \mathbb{R}$$

10 6-15:26

Inverzní funkce

$f:\mathbb{R}\to\mathbb{R}$, $f^{-1}:\mathbb{R}\to\mathbb{R}$

$(f^{-1})'(y) = \frac{1}{f'(x)}$ $\quad y = f(x)$

Jestliže F^{-1} existuje a je diferencovatelná, pak $/D^1$

$F^{-1}\circ F(x) = x$

$(D^1F^{-1})(F(x)) \circ (D^1F)(x) = Id_{\mathbb{R}^n}$

10 6-15:31