Statistics in Computer Science
(Final) Homework 5

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Exercise 1. *Log-normally distributed random variable* $X \sim lnN(\mu, \sigma^2)$ *has probability density function*

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(ln(x)-\mu)^2}{2\sigma^2}\right].$$

Assume that σ *is fixed and find its Fisher information* $\mathcal{I}(\widehat{\mu})$.
We have the following random sample distributed log-normally with $\sigma^2 = 1$

$$4.856, 0.487, 0.580, 0.839, 0.721, 2.416, 0.715, 0.361, 0.703, 5.829$$

Plot scaled log-likelihood function of the sample with μ *on x-axis and scaled log-likelihood* $l^*(\mu|\mathbf{x}) = l(\mu|\mathbf{x}) - \max(l(\mu|\mathbf{x}))$ *on y-axis. Compare* $l^*(\mu|\mathbf{x})$ *with its quadratic approximation calculated using Taylor approximation* $\ln(\frac{L(\mu|\mathbf{x})}{L(\widehat{\mu}|\mathbf{x})}) \approx -\frac{1}{2}\mathcal{I}(\widehat{\mu})(\mu - \widehat{\mu})^2$.

Exercise 2. *Load dataset* one-sample-mean-skull-mf.txt *with variable* skull.L *that represents skull length of ancient Egyptian male population in mm. Let's assume that the skull length is normally distributed, i.e.* $N(\mu, \sigma^2)$.

1. *Test null hypothesis that mean of skull length is equal to 177.568 mm* ($H_0 : \mu = 177.568$) *on significance level* $\alpha = 0.05$.
2. *Calculate* $100 \times (1-\alpha)\%$ *confidence interval for population mean of skull length with* $1 - \alpha = 0.95$.

For (1) use Wald test statistic T_W *and likelihood ratio test statistics* U_{LR}. *For (2) use Wald empirical confidence interval and likelihood ratio confidence interval.*

Exercise 3. *Load dataset* one-sample-mean-skull-mf.txt *with skull lengths of ancient Egyptian population. Calculate location and variability characteristics of skull length of male population. Plot histogram of skull length of male population and overlay it with its kernel density estimation (use* `density(...)` *function). Plot boxplots of skull lengths for males and females.*