IA168 — Problem set 2

Unless otherwise specified, "game" means "two-player extensive-form game with pure strategies only" throughout this problem set.

Problem 1 [3 points]

We say that an imperfect information extensive-form game G is *equivalent* to a strategic form game G' when both have the same number of players and for every player i there is a bijection $f_i : S_i \to S'_i$, such that for every strategy profile $s = (s_1, \ldots, s_n)$ and every player i: $u_i(s) = u'_i(s')$, where $s' = (f_1(s_1), \ldots, f_n(s_n))$.

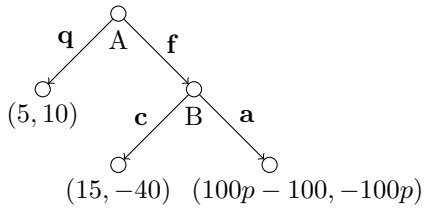
Find an imperfect information extensive-form game that is equivalent to this strategic form game:

	A	B	C
X	(0,1)	(-1,5)	(3,2)
Y	(4,1)	(-3,3)	(2,6)

Problem 2 [5 points]

Consider this real-life situation: Adam is suspicious about the death of Alice, Bob's wife. He confronts Bob, but Bob tells him: "If you keep sticking nose into my bussiness, I will kill you." Adam decides whether to stay **q**uiet or try to **f**orce a confession out of Bob. If he escalates the situation, Bob may either **c**onfess that he killed his wife and turn himself to police or **a**ttack Adam. If Bobs attacks Adam, then with probability $1 - p$ he will win and Adam will be dead and with probability p Adam will be victorious and Bob will not survive this confrontation.

We model this scenario as the perfect-information game depicted below.



In dependence on the parameter p, $0 \leq p \leq 1$, answer the following questions:
How many strategies does each player have?
Which of them are never-best-response?
Which of them are maxmin?
How many strategy profiles are there?
Which of them are Nash equilibria?
Which of them are subgame-perfect equilibria?
Is Bob's threat actually credible?

Problem 3 [7 points]

Find a perfect-information game where all of the following conditions are satisfied:

- there is a strategy profile whose outcome is for both players better than that of any Nash equilibrium;
- there is a Nash equilibrium whose outcome for player 1 is better than that of any subgame-perfect equilibrium;
- there are exactly two subgame-perfect equilibria s, s', and the outcome of s is for both players better than that of s'.

Problem 4 [5 points]

Let G be a perfect-information game. Prove or disprove:

a) For every Nash equilibrium with outcomes (x, y) in G, there exists a subgame-perfect equilibrium with outcomes (x', y') where either $x' \geq x$ or $y' \geq y$.

b) for every Nash equilibrium with outcomes (x, y) in G, there exists a subgame-perfect equilibrium with outcomes (x', y') where $x' \geq x$ and $y' \geq y$.