MA010 Tutorial 2

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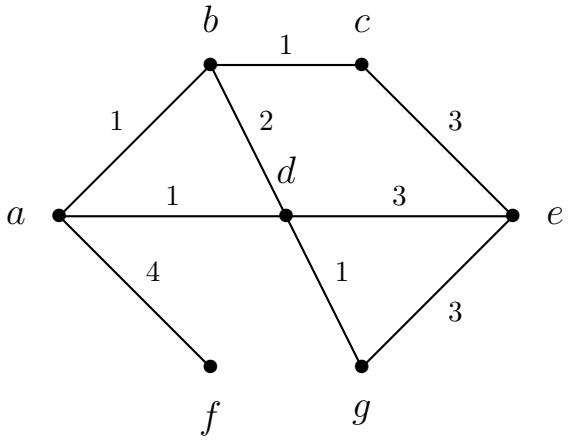
This tutorial covers material from lectures 1 and 2.

Problem 1

Consider the following weighted graph:



(a) Simulate a depth-first search starting at vertex a and looking at adjacent vertices clockwise starting at noon. Draw the resulting search tree.

(b) Simulate a breadth-first search starting at vertex a and looking at adjacent vertices clockwise starting at noon. Draw the resulting search tree.

(c) Find a minimum spanning tree by simulating Jarník's algorithm starting at vertex a and looking at adjacent vertices clockwise starting at noon. Draw the resulting tree.

Problem 2

Let G be a weighted undirected graph, and let T be a minimum spanning tree of G. Suppose that we now create the graph G' from G by increasing the weights of all the edges by some value x.

(a) Is T still a minimum spanning tree in G'? Prove it, or find a counterexample.

(b) Is every shortest path between two vertices in G still the shortest path in G'? Prove it, or find a counterexample.

Source: http://www.cs.umd.edu/class/spring2002/cmsc451/hw2.ps

Problem 3

Show that all minimum spanning trees must have the same multiset of weights.

Source: http://www.eecis.udel.edu/~saunders/courses/621/03f/hwQ.html

Problem 4

(a) Show that in each k-connected graph, each set of k vertices belongs to a common cycle.

(b) For every k, find a k-connected graph with a set of $k+1$ vertices that do not belong to a common cycle.

Source: http://kam.mff.cuni.cz/~jelinek/kag2_z0708/hw2.pdf

Problem 5

Given a directed graph G and a spanning tree T of G, a *forward edge* is an edge not in T that goes from a vertex to one of its descendents in T (it "bypasses" T). Show that if T is a BFS search tree, there are no forward edges.