8.69. v $\mathbb{R}^3$, $M=\{(x,y,z);\ x^2+y^2\le 1,\ z\le x\}$, $z\ge 0$

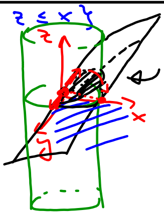
vol M: $\int_M 1\,dx\,dy\,dz = ?$ $\quad (x,y,z) \to (r,\varphi,z)$

$x = r\cos\varphi$, $y = r\sin\varphi$, $z = z$ — JACOBIÁN

$\begin{vmatrix}\cos\varphi & -r\sin\varphi & 0\\ \sin\varphi & r\cos\varphi & 0\\ 0 & 0 & 1\end{vmatrix} = r$

$\int_M dx\,dy\,dz = \int\left(\int\left(\int dz\right)\right) = \int_0^1\int_{-\pi/2}^{\pi/2} r\left(\int_0^{r\cos\varphi} dz\right) d\varphi\, dr = \int_0^1\int_{-\pi/2}^{\pi/2} r^2\cos\varphi\, d\varphi\, dr = \int_0^1 r^2[\sin\varphi]_{-\pi/2}^{\pi/2}\,dr = 2\left[\frac13 r^3\right]_0^1 = \frac23$

$\boxed{\frac23 R\alpha}$ ← $z\le\alpha x$, $x^2+y^2\le R^2$



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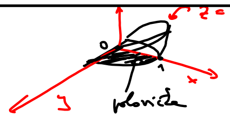
$\int_M dx\,dy\,dz = 2\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^x dz\,dy\,dx = 2\int_0^1\int_0^{\sqrt{1-x^2}} x\,dy\,dx = 2\int_0^1 x\sqrt{1-x^2}\,dx$

$= \int_0^1\sqrt{1-t}\,dt = \left[(1-t)^{3/2}(-1)\frac23\right]_0^1$ $\quad x^2 = t$

$= \frac23$ ✓



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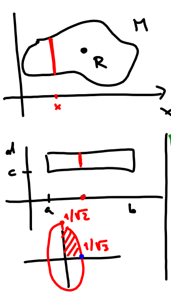

$\iint_M dx\,dy = V$

$\iint_M x\,dx\,dy = T_x$

$\iint_M y\,dx\,dy = T_y$

$R = \frac1V[T_x, T_y]$

8.85 elipsa: $3x^2+2y^2=1$, $x\ge0$, $y\ge0$

$\boxed{V} = \int_0^{1/\sqrt3}\int_0^{\sqrt{\frac12(1-3x^2)}} dy\,dx = \frac{1}{\sqrt2}\int_0^{1/\sqrt3}\sqrt{1-3x^2}\,dx = \frac{1}{\sqrt6}\int_0^{\pi/2}\cos^2 t\,dt$ $\quad y = \sqrt{\frac12(1-3x^2)}$

$\sqrt3 x = \sin t$, $\sqrt3\,dx = \cos t\,dt$

$= \frac{1}{\sqrt6}\left[\frac14\sin 2x + \frac12 x\right]_0^{\pi/2} = \boxed{\frac{\pi}{4\sqrt6}}$

$T_x = \int_0^{1/\sqrt3}\int_0^{\sqrt{\frac12(1-3x^2)}} x\,dy\,dx = \frac{1}{\sqrt2}\int_0^{1/\sqrt3} x\sqrt{1-3x^2}\,dx = \frac{1}{6\sqrt2}\int_0^1\sqrt{1-t}\,dt = \frac{-1}{6\sqrt2}\left[\frac23(1-t)^{3/2}\right]_0^1 = \boxed{\frac{\sqrt2}{18}}$

$3x^2 = t$, $6x\,dx = dt$

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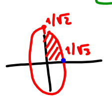
$\int_0^{1/\sqrt3}\int_0^{\sqrt{1/2(1-3x^2)}} y\,dy\,dx = \int_0^{1/\sqrt3}\left[\frac12 y^2\right]_0^{\sqrt{1/2(1-3x^2)}} dx = \frac14\int_0^{1/\sqrt3} 1-3x^2\,dx = \frac{1}{4\sqrt3} - \frac34\left[\frac13 x^3\right]_0^{1/\sqrt3} = \frac{1-1/3}{4\sqrt3} = \boxed{\frac{1}{6\sqrt3}}$



$\iint_M dx\,dy = V$

$\iint_M x\,dx\,dy = T_x$

$\iint_M y\,dx\,dy = T_y$

$R = \frac1V[T_x, T_y]$

8.85 elipsa: $3x^2+2y^2=1$, $x\ge0$, $y\ge0$

$\boxed{V} = \int_0^{1/\sqrt3}\int_0^{\sqrt{\frac12(1-3x^2)}} dy\,dx = \frac{1}{\sqrt2}\int_0^{1/\sqrt3}\sqrt{1-3x^2}\,dx = \frac{1}{\sqrt6}\int_0^{\pi/2}\cos^2 t\,dt$ $\quad y = \sqrt{\frac12(1-3x^2)}$

$\sqrt3 x = \sin t$, $\sqrt3\,dx = \cos t\,dt$

$\left[\frac{4\sqrt3}{9\pi}, \frac{2\sqrt2}{\pi}\right]$

$= \frac{1}{\sqrt6}\left[\frac14\sin 2x + \frac12 x\right]_0^{\pi/2} = \boxed{\frac{\pi}{4\sqrt6}}$

$T_x = \int_0^{1/\sqrt3}\int_0^{\sqrt{\frac12(1-3x^2)}} x\,dy\,dx = \frac{1}{\sqrt2}\int_0^{1/\sqrt3} x\sqrt{1-3x^2}\,dx = \frac{1}{6\sqrt2}\int_0^1\sqrt{1-t}\,dt = \frac{-1}{6\sqrt2}\left[\frac23(1-t)^{3/2}\right]_0^1 = \boxed{\frac{\sqrt2}{18}}$

$3x^2 = t$, $6x\,dx = dt$

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$3x^2+2y^2\le 1$, $x\ge0$, $y\ge0$

$\boxed{\begin{aligned}x &= \tfrac{1}{\sqrt3}r\cos\varphi\\ y &= \tfrac{1}{\sqrt2}r\sin\varphi\end{aligned}}$ $\quad 3x^2+2y^2 = \underbrace{r^2\cos^2\varphi + r^2\sin^2\varphi}_{r^2} \le 1$

$0\le\varphi\le\pi/2$

$V = \int_0^{1}\int_0^{\pi/2}\frac{1}{\sqrt6} r\,d\varphi\,dr = \frac{1}{\sqrt6}\pi/2\cdot\int_0^1 r\,dr = \frac{\pi}{4\sqrt6}$

$T_x = \int_0^1\left(\int_0^{\pi/2}\frac{1}{\sqrt3}r\cos\varphi\frac{1}{\sqrt6}r\,d\varphi\right)dr = \frac{1}{\sqrt{18}}\int_0^1[\sin\varphi]_0^{\pi/2} r^2\,dr$

$= \frac{1}{3\sqrt{18}} = \frac{1}{9\sqrt2} = \frac{\sqrt2}{18}$ ✓

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$G:\mathbb{R}^n\to\mathbb{R}^n$ transformace $\quad \boxed{\int_{G(M)} f\,dx_1\dots dx_n = \int_M f\circ G\,|DG|\,dy_1\dots dy_n}$

$x = G(y)$

k vektorů $\xi_1,\dots,\xi_k$ $\quad (dx_1\wedge\dots\wedge dx_k)(\xi_1,\dots,\xi_k) = |\xi_1\dots\xi_k|$

na $\mathbb{R}^n$ standardní forma objemu $dx_1\wedge\dots\wedge dx_n = \omega_{\mathbb{R}^n}$

$\int_M f(x)\,dx_1\wedge\dots\wedge dx_n$

$\omega\in\Omega^k(\mathbb{R}^n)$ $\quad \boxed{a_1dx_1+a_2dx_2+\dots+a_ndx_n = \alpha}$

$\mathbb{R}^2$

$\boxed{dd = 0}$

f najděte takové $\boxed{\alpha = df}$ $\quad C\subset\mathbb{R}^2$

$\int_C\alpha$ $\quad \boxed{\text{STOKES}}$ $\quad d:\Omega^k\to\Omega^{k+1}$

$\boxed{\int_{\partial M}\alpha = \int_M d\alpha}$

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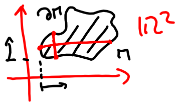

$$\int_{\partial M} \overbrace{-y\,dx + x\,dy}^{\alpha} = 2 \text{ plocha } M$$
$$= \int_M 2\,dx \wedge dy$$

$$d\alpha = -1\, \underset{+1\, dx\wedge dx}{\underset{\shortparallel}{dy \wedge dx}} + dx \wedge dy$$

$$\int_{\partial M} -y\,dx = \text{plocha } M$$

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