Natural Deduction

$$(\mathrm{I}_\top)\ \frac{}{\Gamma \vdash \top} \qquad (\mathrm{Ax})\ \frac{}{\Gamma, \varphi \vdash \varphi}$$

$$(\mathrm{I}_\wedge)\ \frac{\Gamma \vdash \varphi \quad \Delta \vdash \psi}{\Gamma, \Delta \vdash \varphi \wedge \psi} \qquad (\mathrm{E}_\wedge)\ \frac{\Gamma \vdash \varphi \wedge \psi}{\Gamma \vdash \varphi} \quad \frac{\Gamma \vdash \varphi \wedge \psi}{\Gamma \vdash \psi}$$

$$(\mathrm{I}_\vee)\ \frac{\Gamma \vdash \varphi}{\Gamma \vdash \varphi \vee \psi} \quad \frac{\Gamma \vdash \psi}{\Gamma \vdash \varphi \vee \psi} \qquad (\mathrm{E}_\vee)\ \frac{\Gamma \vdash \varphi \vee \psi \quad \Delta, \varphi \vdash \vartheta \quad \Delta', \psi \vdash \vartheta}{\Gamma, \Delta, \Delta' \vdash \vartheta}$$

$$(\mathrm{I}_\neg)\ \frac{\Gamma, \varphi \vdash \bot}{\Gamma \vdash \neg\varphi} \qquad (\mathrm{E}_\neg)\ \frac{\Gamma, \neg\varphi \vdash \bot}{\Gamma \vdash \varphi}$$

$$(\mathrm{I}_\bot)\ \frac{\Gamma \vdash \varphi \quad \Gamma \vdash \neg\varphi}{\Gamma \vdash \bot} \qquad (\mathrm{E}_\bot)\ \frac{\Gamma \vdash \bot}{\Gamma \vdash \varphi}$$

$$(\mathrm{I}_\to)\ \frac{\Gamma, \varphi \vdash \psi}{\Gamma \vdash \varphi \to \psi} \qquad (\mathrm{E}_\to)\ \frac{\Gamma \vdash \varphi \quad \Delta \vdash \varphi \to \psi}{\Gamma, \Delta \vdash \psi}$$

$$(\mathrm{I}_\leftrightarrow)\ \frac{\Gamma, \varphi \vdash \psi \quad \Delta, \psi \vdash \varphi}{\Gamma, \Delta \vdash \varphi \leftrightarrow \psi} \qquad (\mathrm{E}_\leftrightarrow)\ \frac{\Gamma \vdash \varphi \quad \Delta \vdash \varphi \leftrightarrow \psi}{\Gamma, \Delta \vdash \psi} \quad \frac{\Gamma \vdash \psi \quad \Delta \vdash \varphi \leftrightarrow \psi}{\Gamma, \Delta \vdash \varphi}$$

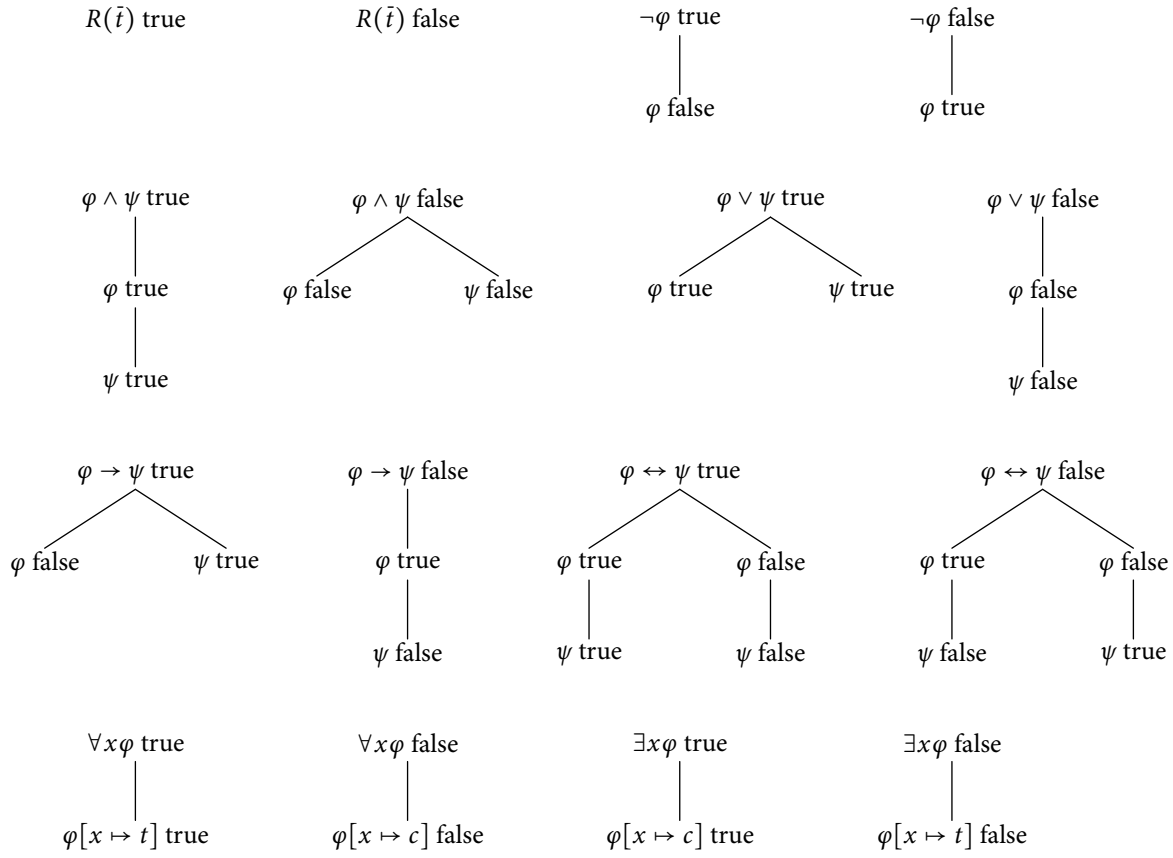
$$(\mathrm{I}_\exists)\ \frac{\Gamma \vdash \varphi[x \mapsto t]}{\Gamma \vdash \exists x \varphi} \qquad (\mathrm{E}_\exists)\ \frac{\Gamma \vdash \exists x \varphi \quad \Delta, \varphi[x \mapsto c] \vdash \psi}{\Gamma, \Delta \vdash \psi}$$

$$(\mathrm{I}_\forall)\ \frac{\Gamma \vdash \varphi[x \mapsto c]}{\Gamma \vdash \forall x \varphi} \qquad (\mathrm{E}_\forall)\ \frac{\Gamma \vdash \forall x \varphi}{\Gamma \vdash \varphi[x \mapsto t]}$$

$$(\mathrm{I}_=)\ \frac{}{\Gamma \vdash t = t} \qquad (\mathrm{E}_=)\ \frac{\Gamma \vdash s = t \quad \Delta \vdash \varphi[x \mapsto s]}{\Gamma, \Delta \vdash \varphi[x \mapsto t]}$$

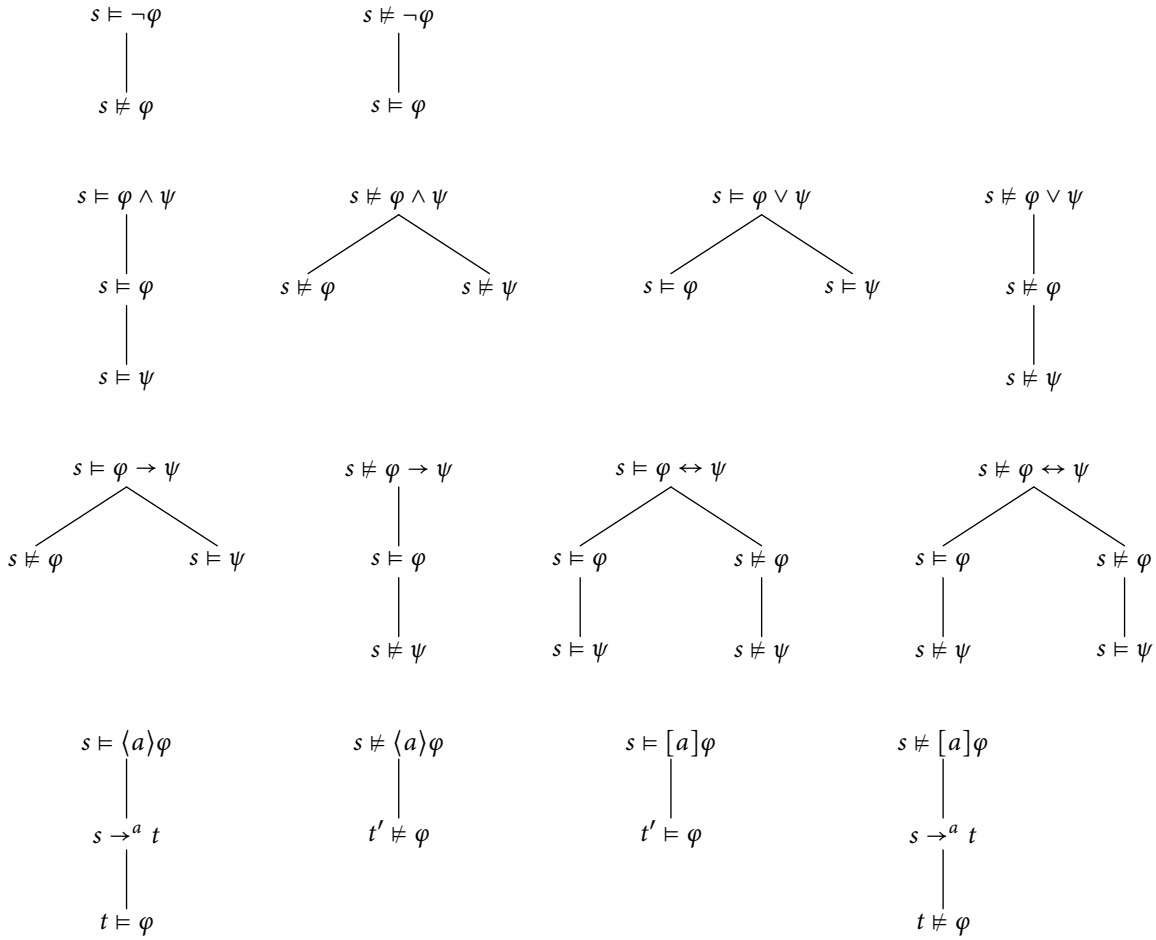
c a *new* constant symbol, s, t arbitrary terms

Tableaux for First-Order Logic



c a new constant symbol, t an arbitrary term

Tableaux for Modal Logic



t a new state, t' every state with entry $s \to^a t'$ on the branch