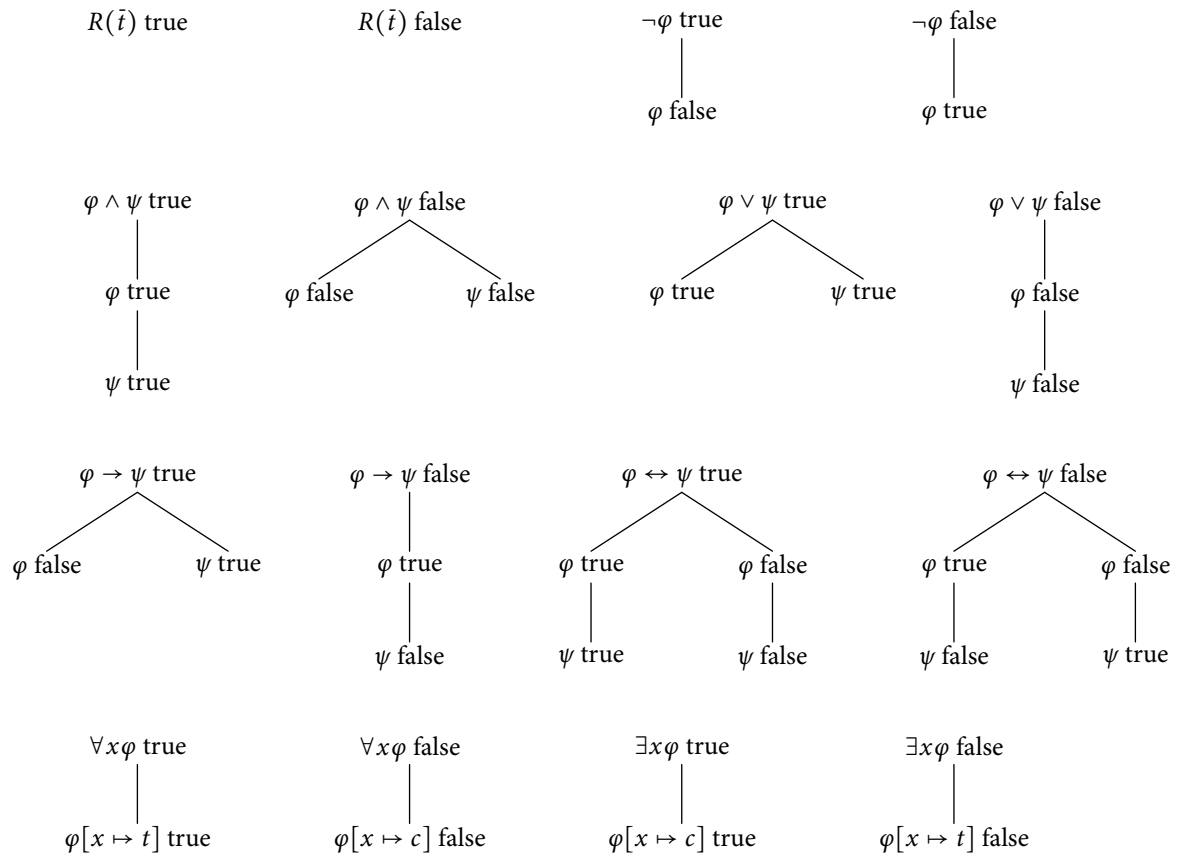


## Natural Deduction

$$\begin{array}{l}
 (I_{\top}) \frac{}{\Gamma \vdash \top} \\
 (I_{\wedge}) \frac{\Gamma \vdash \varphi \quad \Delta \vdash \psi}{\Gamma, \Delta \vdash \varphi \wedge \psi} \\
 (I_{\vee}) \frac{\Gamma \vdash \varphi}{\Gamma \vdash \varphi \vee \psi} \quad \frac{\Gamma \vdash \psi}{\Gamma \vdash \varphi \vee \psi} \\
 (I_{\neg}) \frac{\Gamma, \varphi \vdash \perp}{\Gamma \vdash \neg \varphi} \\
 (I_{\perp}) \frac{\Gamma \vdash \varphi \quad \Gamma \vdash \neg \varphi}{\Gamma \vdash \perp} \\
 (I_{\rightarrow}) \frac{\Gamma, \varphi \vdash \psi}{\Gamma \vdash \varphi \rightarrow \psi} \\
 (I_{\leftrightarrow}) \frac{\Gamma, \varphi \vdash \psi \quad \Delta, \psi \vdash \varphi}{\Gamma, \Delta \vdash \varphi \leftrightarrow \psi} \\
 (I_{\exists}) \frac{\Gamma \vdash \varphi[x \mapsto t]}{\Gamma \vdash \exists x \varphi} \\
 (I_{\forall}) \frac{\Gamma \vdash \varphi[x \mapsto c]}{\Gamma \vdash \forall x \varphi} \\
 (I_{=}) \frac{}{\Gamma \vdash t = t}
 \end{array}
 \qquad
 \begin{array}{l}
 (Ax) \frac{}{\Gamma, \varphi \vdash \varphi} \\
 (E_{\wedge}) \frac{\Gamma \vdash \varphi \wedge \psi}{\Gamma \vdash \varphi} \quad \frac{\Gamma \vdash \varphi \wedge \psi}{\Gamma \vdash \psi} \\
 (E_{\vee}) \frac{\Gamma \vdash \varphi \vee \psi \quad \Delta, \varphi \vdash \vartheta \quad \Delta', \psi \vdash \vartheta}{\Gamma, \Delta, \Delta' \vdash \vartheta} \\
 (E_{\neg}) \frac{\Gamma, \neg \varphi \vdash \perp}{\Gamma \vdash \varphi} \\
 (E_{\perp}) \frac{\Gamma \vdash \perp}{\Gamma \vdash \varphi} \\
 (E_{\rightarrow}) \frac{\Gamma \vdash \varphi \quad \Delta \vdash \varphi \rightarrow \psi}{\Gamma, \Delta \vdash \psi} \\
 (E_{\leftrightarrow}) \frac{\Gamma \vdash \varphi \quad \Delta \vdash \varphi \leftrightarrow \psi}{\Gamma, \Delta \vdash \psi} \quad \frac{\Gamma \vdash \psi \quad \Delta \vdash \varphi \leftrightarrow \psi}{\Gamma, \Delta \vdash \varphi} \\
 (E_{\exists}) \frac{\Gamma \vdash \exists x \varphi \quad \Delta, \varphi[x \mapsto c] \vdash \psi}{\Gamma, \Delta \vdash \psi} \\
 (E_{\forall}) \frac{\Gamma \vdash \forall x \varphi}{\Gamma \vdash \varphi[x \mapsto t]} \\
 (E_{=}) \frac{\Gamma \vdash s = t \quad \Delta \vdash \varphi[x \mapsto s]}{\Gamma, \Delta \vdash \varphi[x \mapsto t]}
 \end{array}$$

$c$  a new constant symbol,  $s, t$  arbitrary terms

# Tableaux for First-Order Logic



$c$  a new constant symbol,  $t$  an arbitrary term

## Tableaux for Modal Logic

$$\begin{array}{c} s \models \neg\varphi \\ | \\ s \not\models \varphi \end{array}$$

$$\begin{array}{c} s \not\models \neg\varphi \\ | \\ s \models \varphi \end{array}$$

$$\begin{array}{c} s \models \varphi \wedge \psi \\ | \\ s \models \varphi \\ | \\ s \models \psi \end{array}$$

$$\begin{array}{c} s \not\models \varphi \wedge \psi \\ / \quad \backslash \\ s \not\models \varphi \quad s \not\models \psi \end{array}$$

$$\begin{array}{c} s \models \varphi \vee \psi \\ / \quad \backslash \\ s \models \varphi \quad s \models \psi \end{array}$$

$$\begin{array}{c} s \not\models \varphi \vee \psi \\ | \\ s \not\models \varphi \\ | \\ s \not\models \psi \end{array}$$

$$\begin{array}{c} s \models \varphi \rightarrow \psi \\ / \quad \backslash \\ s \not\models \varphi \quad s \models \psi \end{array}$$

$$\begin{array}{c} s \not\models \varphi \rightarrow \psi \\ | \\ s \models \varphi \\ | \\ s \not\models \psi \end{array}$$

$$\begin{array}{c} s \models \varphi \leftrightarrow \psi \\ / \quad \backslash \\ s \models \varphi \quad s \not\models \varphi \\ | \quad \quad | \\ s \models \psi \quad s \not\models \psi \end{array}$$

$$\begin{array}{c} s \not\models \varphi \leftrightarrow \psi \\ / \quad \backslash \\ s \models \varphi \quad s \not\models \varphi \\ | \quad \quad | \\ s \not\models \psi \quad s \models \psi \end{array}$$

$$\begin{array}{c} s \models \langle a \rangle \varphi \\ | \\ s \rightarrow^a t \\ | \\ t \models \varphi \end{array}$$

$$\begin{array}{c} s \not\models \langle a \rangle \varphi \\ | \\ t' \not\models \varphi \end{array}$$

$$\begin{array}{c} s \models [a] \varphi \\ | \\ t' \models \varphi \end{array}$$

$$\begin{array}{c} s \not\models [a] \varphi \\ | \\ s \rightarrow^a t \\ | \\ t \not\models \varphi \end{array}$$

$t$  a new state,  $t'$  every state with entry  $s \rightarrow^a t'$  on the branch