Exercise 1 Suppose we are given a predicate flight(*From*, *To*, *Time*, *Price*) containing information about direct flights including the starting airport, the destination, the flight time, and the price of a ticket. Write a Prolog program computing a predicate travel(*From*, *To*, *Stops*, *Time*, *Price*) indicating all possibilities to travel from one city to another using one or several flights.

Exercise 2 Write a Prolog predicate fib(N, X) computing the Fibonacci sequence. Evaluate fib($3, X$) and fib($N, 5$).

Exercise 3 Write Prolog definitions of the following predicates.

length(*List*, N)	N is the length of *List* .
reverse(X, Y)	Y is the reverse of the list X .
append(X, Y, Z)	Z is the concatenation of the lists X and Y .
map(X, Y)	maps a list $X = [X_1, \ldots, X_n]$ to $Y = [f(X_1), \ldots, f(X_n)]$.
fold_left(X, Y, Z)	maps $Y = [Y_1, \ldots, Y_n]$ to $Z = f(\cdots f(f(X, Y_1), Y_2) \cdots, Y_n)$.
fold_right(X, Y, Z)	maps $Y = [Y_1, \ldots, Y_n]$ to $Z = f(Y_1, f(Y_2, \ldots, f(Y_n, X) \ldots)))$.

Exercise 4 Write a naive sort function

```
naive_sort(X,Y) :- permute(X,Y), sorted(Y).
```

by implementing the relations

sorted(X)	checks that the list X is sorted.
insert(X, Y, Z)	if the list Z is obtained from Y by inserting X at an arbitrary position.
permute(X, Y)	if the list Y is a permutation of X .

Implement merge sort using a relation

merge(X, Y, Z)	merges two sorted lists X and Y into Z .

Exercise 5 We consider directed graphs of the form $\langle V, E \rangle$. Express the following relation in relational algebra.

(a) x and y are not connected by an edge.

(b) The edge $\langle x, y \rangle$ is part of a triangle.

(c) x has at least two neighbours.

(d) Every neighbour of x is also a neighbour of y.

Exercise 6 Evaluate the following Datalog program on the tree $\langle V, E, P \rangle$ to the right.

$$
\begin{aligned}
U &\leftarrow S(x,y) \wedge W(x) \wedge W(y) \\
W(x) &\leftarrow P(x) \\
W(x) &\leftarrow E(x,y) \wedge W(y) \\
S(x,y) &\leftarrow E(z,x) \wedge E(z,y) \wedge x \neq y \\
R(x,y) &\leftarrow P(x) \wedge x = y \\
R(x,y) &\leftarrow E(x,z) \wedge R(z,y) \\
R(x,y) &\leftarrow R(x,z) \wedge E(z,y)
\end{aligned}
$$