IA168 — Problem set 1

Throughout this problem set, "game" means "two-player strategic-form game with pure strategies only".

Problem 1 [6 points]

Find a game with exactly 2 **Pareto optimal** strategy profiles and exactly 2 **Nash equilibria** so that:

a) both of the Nash equilibria are Pareto optimal;

b) exactly one Nash equilibrium is Pareto optimal;

c) neither of the Nash equilibria is Pareto optimal.

Problem 2 [10 points]

Consider the following zero-sum game, defined by the payoff table for player 1:

	A_2	B_2
A_1	x	y^2
B_1	y	x^2

where the payoffs of player 2 are the opposite values of these payoffs (e.g. $u_2(A_1, A_2) = -u_1(A_1, A_2) = -x$) and $x, y \in \mathbb{R}$.

Player 1 and player 2 will play this game infinitely many times and we will denote by $s_{1,i} \in \{A_1, B_1\}$ the strategy that player 1 chooses in the i-th iteration, by $s_{2,i} \in \{A_2, B_2\}$ the strategy that player 2 chooses in the i-th iteration and by s_i the strategy profile $(s_{1,i}, s_{2,i})$.

Suppose that $s_{1,1} = A_1$, $s_{2,1} = A_2$, $s_{1,i} = A_1$ iff A_1 is a best response to $s_{2,i-1}$ and $s_{2,i} = A_2$ iff A_2 is a best response to $s_{1,i-1}$ for $i > 1$. Find the necessary and sufficient condition for x, y so that:

a) $\exists i \in \mathbb{N}$: $s_i = s_{i+1} = (A_1, A_2)$

b) $\exists i \in \mathbb{N}$: $s_i = s_{i+1} = (B_1, B_2)$

c) $\forall i \in \mathbb{N}$: $s_i \neq s_{i+1}$, $s_i \neq s_{i+2}$ and $s_i \neq s_{i+3}$

d) $\exists i \in \mathbb{N}$: $s_i \neq s_{i+1}$ and either $s_i = s_{i+2}$ or $s_i = s_{i+3}$

Explain your reasoning.

Problem 3 [4 points]

Prove that each Nash equilibrium is rationalizable and survives IESDS (Theorem 17.2).