IA168 — Problem set 2

Throughout this problem set, “game” means “two-player strategic-form game with mixed strategies”.

Problem 1 [7 points]

Consider a game where each player has exactly five pure strategies, called A_i, B_i, C_i, D_i, E_i for $i \in \{1, 2\}$. The utility functions are defined by the following table:

	A_2	B_2	C_2	D_2	E_2
A_1	$(-4, 4)$	$(-3, 2)$	$(2, 2)$	$(4, 1)$	$(-2, -1)$
B_1	$(0, 6)$	$(-3, 3)$	$(2, 3)$	$(7, 3)$	$(-3, 3)$
C_1	$(-6, 0)$	$(3, 6)$	$(4, 1)$	$(1, 2)$	$(-6, 0)$
D_1	$(-2, -1)$	$(-2, 7)$	$(2, 2)$	$(5, 3)$	$(-5, 3)$
E_1	$(-6, 3)$	$(-6, 3)$	$(1, 3)$	$(3, 2)$	$(3, 6)$

(a) Find a Nash equilibrium $\sigma^* = (\sigma_1^*, \sigma_2^*)$ such that $|\operatorname{supp}(\sigma_1^*)| + |\operatorname{supp}(\sigma_2^*)|$ is maximal.

(b) Prove that σ^* is a Nash equilibrium.

(c) Prove the maximality of $|\operatorname{supp}(\sigma_1^*)| + |\operatorname{supp}(\sigma_2^*)|$.

Problem 2 [5 points]

Give an example of a game where

(a) there is no weakly dominating pure strategy, but there exists a weakly dominating mixed strategy;

(b) there is no weakly dominating pure strategy, but there exists a very weakly dominating mixed strategy;

(c) there is no strictly dominated pure strategy, but there exists a strictly dominated mixed strategy;

(d) there is no very weakly dominated pure strategy, but there exists a strictly dominated mixed strategy

or prove that no such game exists.

Problem 3 [8 points]

Prove that for every $k \in M$ there is a game with exactly k Nash equilibria, where

(a) $M = \{2^n - 1 \mid n \in \mathbb{N}\}$;

(b) $M = \mathbb{N}$.