

Existence and God

Pavel Tichý

Journal of Philosophy, 1979

Marie Duží

VSB-Technical University Ostrava

marie.duzi@vsb.cz

Prof. Dr. Pavel Tichý

- 1954 – 1959: Charles University
 - Philosophy and Mathematics
- 1968: left for England (Exeter)
- 1971: emigrated to New Zealand, Otago, Prof. University of Dunedin
- October 26, 1994: died in Dunedin

The contents

1. Denotation and existence (in general)
2. Extensions vs. intensions;
intensional essentialism → requisites
3. Descart's proof: two errors
4. Anselm's argument: logically correct,
(doubts about the validity of a premise)

Denotation and existence

- Jimmy Carter has blue eyes
 - *How to evaluate*: take the individual and examine whether he has the property of having blue eyes
- Jimmy Carter *exists*
 - Is existence just another feature that can be ascribed to *individuals*?
 - How should we evaluate such an existential statement?
 - Take the individual → but then it trivially exists!
 - ***The set of individuals is given a priori*** (a „pre-concept“ of the universe in TIL)
- Hence every sentence claiming the existence of an individual is ***necessarily true***
- (as many philosophers and logicians observed; Hintikka, Kant, ...)

“Existence is not a property of things”

- Aristoteles, Kant, Russell, ...
- **But:** Existence *is* non-trivially predicted and coherently denied of something
- *True and informative sentences:*
 - *The President of the USA exists*
 - *The King of France does not exist*
- Do these sentences ascribe existence to any individual?
- But Obama, or Trump, or whoever, is not mentioned here; and *which* individual would be mentioned by the second sentence?

The King of France does not exist

- What does it predict the *non-existence* about?
- About “non-existing individual”?
- We don’t deal with possibilities (Parmenides)
- The subject to which non-existence is ascribed is an individual *office, role, ‘thing to be’*:

- **Properly partial function** (mapping):

$\omega \rightarrow (\tau \rightarrow \iota); \iota_{\tau\omega};$

ω - modal parameter (possible worlds); τ - temporal parameter (times); ι - universe of discourse (individuals)

$\langle w_1, t_1 \rangle \rightarrow \text{Ind}_1$

$\langle w_2, t_2 \rangle \rightarrow \text{Ind}_2$

$\langle w_i, t_i \rangle \rightarrow \text{nothing (value gap)}$

$\langle w_k, t_l \rangle \rightarrow \text{Ind}_3$

...

Offices, roles, things to be

- **Occupied** – there is an individual playing the role
(President of Zimbabwe, the Pope, the first man who run 100 m under 10 seconds, ...)
- **Vacant** – no individual playing the role
(the King of France, the first man to run 100 m under 9 seconds, ...)

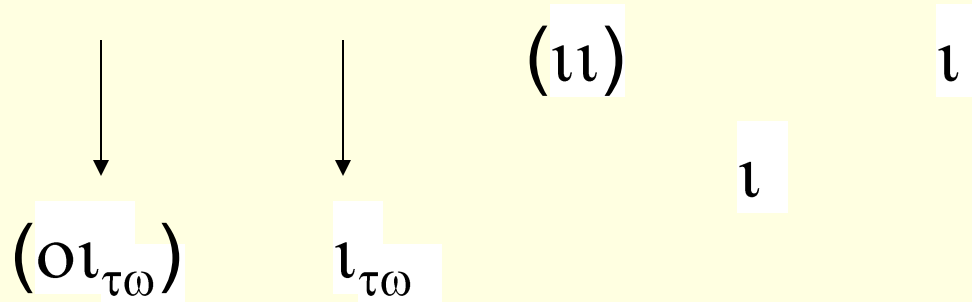
Existence *is* a property

- Not of individuals, but
- of ***intensions*** (or functions, in general): the property of being *occupied, instanciated, having a value*, in a given *w*(orld) at *t*(ime)

■ $Exist / (o_{\tau\omega})_{\tau\omega}$

■ ***The President of USA exists:***

■ $\lambda w \lambda t [{}^0Exist_{wt} \lambda w \lambda t [{}^0President_{wt} {}^0USA]]$



Existence concerns *intensions*

$\lambda w \lambda t [{}^0\text{Exist}_{wt} \underbrace{\lambda w \lambda t [{}^0\text{President}_{wt} {}^0\text{USA}]}_{\text{de dicto}}]$

de dicto

$\lambda w \lambda t [{}^0\text{Blue-eyed}_{wt} \underbrace{\lambda w \lambda t [{}^0\text{President}_{wt} {}^0\text{USA}]_{wt}}_{\text{de re}}]$

de re

Definition of existence

- ${}^0\text{Exist}^1 = \lambda w \lambda t \lambda p [{}^0\exists \lambda x [p_{wt} x]], p \rightarrow (o\alpha)_{\tau\omega}$
- ${}^0\text{Exist}^2 = \lambda w \lambda t \lambda u [{}^0\exists \lambda x [u_{wt} = x]], u \rightarrow \alpha_{\tau\omega}$

$$\lambda w \lambda t [{}^0\text{Exist}^2_{wt} \lambda w \lambda t [{}^0\text{President}_{wt} {}^0\text{USA}]] =_{\text{df}}$$

$$\lambda w \lambda t [\lambda w \lambda t \lambda u [{}^0\exists \lambda x [u_{wt} = x]]_{wt}$$

$$\lambda w \lambda t [{}^0\text{President}_{wt} {}^0\text{USA}]] =_{\beta}$$

$$\lambda w \lambda t [{}^0\exists \lambda x [\lambda w \lambda t [{}^0\text{President}_{wt} {}^0\text{USA}]]_{wt} = x]]$$

(β -reduction)

Requisites of an office

- Property R is a requisite of an office U :
 - **Necessarily**,
if a occupies the office $U \rightarrow \iota_{\tau\omega}$, then a has the property $R \rightarrow (\circ\iota)_{\tau\omega}$
- $[{}^0\text{Req } R \ U] =$
 $\forall w \forall t [[{}^0\text{Exist}_{wt} \ U] \supset [{}^0\text{True}_{wt} \ \lambda w \lambda t [R_{wt} \ U_{wt}]]]$
- *Example.* Requisites of the President of USA: to be a human being, properly elected, inaugurated, US citizenship, ...

Valid/invalid argument

■ *Invalid:*

(+) R is a requisite of $U \Rightarrow$
the holder of U has the property R

$[{}^0Req R U] \Rightarrow \lambda w \lambda t [R_{wt} U_{wt}]$: the so-constructed proposition *is not true* if U_{wt} fails – truth-value gap !!

Yet the requisite relation obtains between intensions independently of contingent empirical facts

■ *Valid:*

(++) the office U is occupied;
 R is a requisite of $U \Rightarrow$
the holder of U has the property R

$[{}^0Req R U] \& \lambda w \lambda t [{}^0Exist_{wt} U] \Rightarrow \lambda w \lambda t [R_{wt} U_{wt}]$

Valid inference rule (++)

- $[{}^0Req R U] =$
 $\forall w \forall t [[{}^0Exist_{wt} U] \supset [{}^0True_{wt} \lambda w \lambda t [R_{wt} U_{wt}]]]$

$$\begin{array}{c}
 (++) \quad \frac{[{}^0Req R I] \wedge [{}^0Exist_{wt} U_{wt}]}{[R_{wt} U_{wt}]}
 \end{array}$$

Example:

$$[{}^0Req \ {}^0Politician \ \lambda w \lambda t \ [{}^0President_{wt} \ {}^0USA]]$$

Higher-level offices

- The President of USA / $l_{\tau\omega}$
 - occupied by individuals; **level 1**
- The highest executive office of USA
 - occupied by **offices**:
currently and actually by the President (rather than the King) of the USA
 - Entity / $(l_{\tau\omega})_{\tau\omega}$ — **level 2**
- The most favourite proposition of A. Eistein / $(o_{\tau\omega})_{\tau\omega}$

What does ‘*knowing* an office’ amount for?

- We can never know the uncountable infinite mapping
- Only that we do understand an ***instruction***:
 - ‘**the Mayor of Dunedin**’; knowing the *meaning* amounts for knowing the instruction how to evaluate in *any* possible world w (λw) at *any* time t (λt) the procedure
$$[{}^0\text{Mayor-of}_{wt} {}^0\text{Dunedin}]$$
 - It does *not mean to be able* to execute, nor to know the holder!
 - **We can never know the actual world**

What does 'God' denote ?

- If 'God' denoted an individual ($\rightarrow \iota$), than it'd be purely contingent matter whether he is omnipotent, omniscient, benevolent, ..., because any individual might have been malevolent
- 'God' denotes a thing to be: individual office ($\rightarrow \iota_{\tau\omega}$)
- Requisites of the office: all positives
- Question: "Does God exist?" is reasonable; we ask whether the God-office is occupied

Descartes' (ontological) proof

- René Descartes (1596 – 1650)
- The essence of the God-idea are all the positive properties

- Invalid scenario:

God has all the positives; existence is a positive. Hence, God exists.

What is wrong here?

Descartes' proof: two problems

1. Existence is a requisite of the God-office.
 - **But:** requisites of an individual office are properties of individuals \Rightarrow existence would have to be such a property (which is not the case)
2. He applies the invalid inference rule (+):
 - Existence is a requisite of the God-office \Rightarrow the holder of the office has the property of existence (**missing assumption:** *if the office is occupied*)

Schema of modal ontological proofs

- **Essence of God** $G = \{P_1, \dots, P_n\}$, where $P_i / (o1)_{\tau\omega}$ are all the positives
- Hence, **(analytically) necessarily** (*ex definitione*) God has all the positive properties (if he can have them, if they do not contradict each other):
 - ◆ $P_i(G) \rightarrow$
 - $(\text{◆ } P_i(G) \rightarrow \text{■ } P_i(G)) = (\text{◆ } P_i(G) \rightarrow \text{■ } P_i(G))$
- Existence E is positive, and it is **possible** that God exists (the concept of God does not involve a contradiction): ◆ $E(G)$.
- Hence, ■ $E(G)$
- **God exists, necessarily.**

What is wrong here?

- (regardless of the problem of existence ascribed to individuals)
- Invalid inference (+)
 - from necessary *intensional (de dicto)* truth (*ex definitione*) to *extensional (de re)* truth

It is necessary that the King of France is a king
(*de dicto*)

Possibly the King of France is a king. (*de dicto*)

⇒ *The King of France is necessarily a king.* (*de re*)

Modality *de dicto* / *de re*

- The King of France might not be a king
 - **“Almost” (analytically) true:**
 - No individual is necessarily a king!
 - **If FK exists, then FK might not be a king:**
 $FK \rightarrow \iota_{\tau\omega}, X \rightarrow \iota.$
 - $\lambda w \lambda t [\lambda x [\exists w^* \exists t^* \neg [FK_{w^*t^*} = x]] FK_{wt}]$ *de re*
- It is possible that the King of France is not a king
 - **Analytically false**
 - $\lambda w \lambda t [\exists w^* \exists t^* \neg [FK_{w^*t^*} FK_{w^*t^*}]]$ *de dicto*

Modality de dicto / de re

- Analogicky
- Dřevěné stoly jsou nutně dřevěné
 - Nepravda!
- Nutně, dřevěné stoly jsou dřevěné
 - Pravda!
- (analyzujte)

Anselm's ontological arguments

- St. Anselm of Canterbury (1033 – 1109)
- ***Proslogion II*** – complicated argument following in principle the flawed schema of modal ontological proofs
- ***Proslogion III*** (prayer) – simple, elegant and transparent argument which had been neglected

Anselm's ontological argument:

‘Thy to whom nothing greater is conceivable’, exist so truly and actually that it is not thinkable that you would not exist.

For it is thinkable that there is something the non-existence of which is not thinkable;

And this thing is greater than anything the non-existence of which is thinkable.

Hence, if it were thinkable that ‘Thy to whom nothing greater is conceivable’ might not exist, then ‘Thy to whom nothing greater is conceivable’ would not be ‘Thy to whom nothing greater is conceivable’ – contradiction

‘Thy to whom nothing greater is conceivable’

- Descartes assumes that he knows the essence of God
- St. Anselm is far from such arrogance
- He addresses God by the modest:
 - ‘Thy to whom nothing greater is conceivable’
- Let’s analyze this description

‘Thy to whom nothing greater is conceivable’

‘greater than’ → binary relation (-in intension) between what ?

Charles is greater than Peter: „greater what“ ???

A. The office of the President of USA is greater than the office of the richest peanut farmer in Georgia, but

B. Jimmy Carter is not greater than Jimmy Carter

Hence: the *concepts* of both offices in (A) *de dicto* (substitution test, not in B !)

Greater than → $(o_{\tau\omega} l_{\tau\omega})_{\tau\omega}$ the relation-in-intension between **offices**

‘Thy to whom nothing greater is conceivable’

- ‘Thy to whom nothing greater is conceivable’
– abbr. $NV \rightarrow (1_{\tau\omega})_{\tau\omega}$
- NV is the second-level office
- God-office occupies NV
- Anselm (even a fool) understand NV , they have a concept of NV ; what does it amount for an individual office to occupy NV
- But they do not know what does it amount for an individual to occupy God-office

‘Thy to whom nothing greater is conceivable’ (*NV*)

- Requisites of *NV* are properties of individual offices
- Existence is a property of individual offices
- Hence – existence can be a requisite of the office *NV*
- *Necessary existence*: to be occupied in all possible worlds (and times)

Anselm's principles

(A) Individual office that has the property of necessary existence is **greater than** any other office lacking this property

(B) Necessary existence is a requisite of *NV*

Anselm did not apply the invalid inference (+) –
slide 11

Anselm's assumptions

- (i) There is an individual office with necessary existence (Anselm considered to be trivially valid; Tichý proved)
- (ii) **An office with necessary existence is greater than the office without**
- (iii) The office *NV* is occupied (O):
(Anselm proves it)

ad i) –

there is a necessarily occupied office

■ Tichý:

- Defined the second-level office L, which has the property of necessary existence:
- Office L of ***the smallest occupied individual office***

Anselm's valid argument

- Office *NV* has necessary existence among its requisites,
- Office *NV* is occupied,
- ⇒ Hence God-office that occupies *NV* has the property of necessary existence
- ⇒ ***God exists, necessarily.***

The assumption that *NV is occupied*

- There are maximal offices to which greater are not conceivable
- *Reductio ad absurdum*: if there were two such things, then these things would have to be greater *via different things*; but then it would be conceivable that there is a thing that has both the things *via* which it is greater
- Conclusion:
God necessarily exists, hence also actually?

Gaunilo's objection

- By the same line of reasoning it is possible to prove the existence of the most perfect island
- The office *NVI* – the greatest island office – does not have the requisite of necessary existence
- Even if it were true that in the actual world islands always existed and will exist, there is nothing impossible in conceiving a world void of islands

Anselm's key assumption

- (slide 25): the office with necessary existence is greater than any other office
- ***Is necessarily occupied office eo ipso greater than any other office which is conceivably vacant?***
- *The greater (more important) office, the more difficult it is to occupy it;*

Anselm's key assumption

- The first man to run 100 m under 9 s
is greater than (i.e. less occupied)
- The first man who run 100 m under 10 s
(*Carl Lewis, 9.86, Usain Bolt 9.58*)

Tichý – necessarily occupied office L: the
smallest ...

Is the office of the most rotten apple core in
Chicago litter basket greater than the office of
the Pope ???

Anselm's key assumption: weaker one

- (A') to any office that is *not* necessarily occupied there is a *greater* necessarily occupied office
- (A') and (O) suffice to prove God's existence
- The first man who run 100 m under 9 s
 - It does not have necessary existence
 - Let us extend the office: in each w, t where the office is vacant let's assign *some* individual as the value
 - **But: in this way we weaken the office** – it will not have the desirable requisites – we obtain necessarily occupied office, but the price is too high. We lose the greatness; lacking better candidates it will be occupied by lower-quality individuals

Necessary existence and greatness

- Contra Anselm
 - **Necessarily occupied is *contra* greatness**
- L* (the opposite of L): the office of the greatest occupied office
- L* is not particularly great: among its requisites there is no perfect quality; its essence is really poor
 - The more frequently occupied office, the smaller its essence
 - There are worlds in which everybody is ignorant of this or that fact: in this world L* must be occupied by a low-quality individual; simply because there are no better candidates

Proposition and truth

- The **more** generally (in more w,t) a *proposition is true*, the **less** informative it is
- Necessarily true proposition is informationally valueless (concerning an *empirical information*)
- Necessarily occupied individual office is boring and grey; it does not demand much of an individual to occupy it

Tichý:

This analogy is even more accurate than it might seem.

Individual office has the property of necessary existence iff the proposition that this office is occupied is true in all worlds and times.

If God existed necessarily then the proposition that God exists were a tautology.

It is difficult and hardly acceptable to suppose that a believer differs from an atheist by the fact that the believer assents to a tautology