

Exercise 1 Suppose we are given a predicate $\text{flight}(\text{From}, \text{To}, \text{Time}, \text{Price})$ containing information about direct flights including the starting airport, the destination, the flight time, and the price of a ticket. Write a Prolog program computing a predicate $\text{travel}(\text{From}, \text{To}, \text{Stops}, \text{Time}, \text{Price})$ indicating all possibilities to travel from one city to another using one or several flights.

Exercise 2 Write a Prolog predicate $\text{fib}(N, X)$ computing the Fibonacci sequence. Evaluate $\text{fib}(3, X)$ and $\text{fib}(N, 5)$.

Exercise 3 Write Prolog definitions of the following predicates.

$\text{length}(\text{List}, N)$	N is the length of List .
$\text{append}(X, Y, Z)$	Z is the concatenation of the lists X and Y .
$\text{reverse}(X, Y)$	Y is the reverse of the list X .
$\text{map}(X, Y)$	maps a list $X = [X_1, \dots, X_n]$ to $Y = [f(X_1), \dots, f(X_n)]$.
$\text{fold_left}(X, Y, Z)$	maps $Y = [Y_1, \dots, Y_n]$ to $Z = f(\dots f(f(X, Y_1), Y_2) \dots, Y_n)$.
$\text{fold_right}(X, Y, Z)$	maps $Y = [Y_1, \dots, Y_n]$ to $Z = f(Y_1, f(Y_2, \dots, f(Y_n, X) \dots))$.

The Prolog notation for lists is as follows:

$[]$ $[X, Y, Z]$ $[X|Y]$ $[X, Y|Z]$.

Exercise 4 Write a naive sort function

$\text{naive_sort}(X, Y) :- \text{permute}(X, Y), \text{sorted}(Y)$.

by implementing the relations

$\text{sorted}(X)$	checks that the list X is sorted.
$\text{insert}(X, Y, Z)$	if the list Z is obtained from Y by inserting X at an arbitrary position.
$\text{permute}(X, Y)$	if the list Y is a permutation of X .

Implement merge sort using a relation

$\text{merge}(X, Y, Z)$	merges two sorted lists X and Y into Z .
$\text{split}(X, Y, Z)$	splits the list X into two lists Y and Z .

Exercise 5 We consider directed graphs of the form $\langle V, E \rangle$. Express the following relation in relational algebra.

- x and y are not connected by an edge.
- The edge $\langle x, y \rangle$ is part of a triangle.
- x has at least two neighbours.
- Every neighbour of x is also a neighbour of y .

Exercise 6 Evaluate the following Datalog program on the tree $\langle V, E, P \rangle$ to the right.

$U \leftarrow S(x, y) \wedge W(x) \wedge W(y)$
 $W(x) \leftarrow P(x)$
 $W(x) \leftarrow E(x, y) \wedge W(y)$
 $S(x, y) \leftarrow E(z, x) \wedge E(z, y) \wedge x \neq y$
 $R(x, y) \leftarrow P(x) \wedge x = y$
 $R(x, y) \leftarrow E(x, z) \wedge R(z, y)$
 $R(x, y) \leftarrow R(x, z) \wedge E(z, y)$

