Exercise 1 Which of the following implications are valid in intuitionistic logic? Give either a tableau proof or a counterexample.

(a) $\varphi \wedge \psi \to \psi \wedge \varphi$

(b) $\psi \to ((\varphi \wedge \psi) \vee \psi)$

(c) $(\neg\psi \to \neg\varphi) \to (\varphi \to \psi)$

(d) $\varphi \to \neg\neg\varphi$

(e) $((\varphi \wedge \psi) \vee \psi) \to \psi$

(f) $\neg\neg\varphi \to \varphi$

(g) $\varphi \vee \neg\varphi$

(h) $\neg(\neg\varphi \wedge \neg\psi) \to (\varphi \vee \psi)$ (This one is tricky.)

(i) $\varphi \to \exists x \varphi$

(j) $\forall x \varphi \to \varphi$

(k) $\forall x R(x, x) \to \forall x \exists y R(f(x), y)$

(l) $\exists x(\varphi \vee \psi) \to \exists x \varphi \vee \exists x \psi$

(m) $\exists x \varphi \vee \exists x \psi \to \exists x(\varphi \vee \psi)$

(n) $\forall x \varphi \wedge \forall x \psi \to \forall x(\varphi \wedge \psi)$

(o) $\forall x(\varphi \wedge \psi) \to \forall x \varphi \wedge \forall x \psi$

(p) $\forall x \forall y[\varphi(x) \leftrightarrow \varphi(y)] \wedge \exists x \varphi(x) \to \forall x \varphi(x)$