Course Control and System Theory of Rational Systems Motivated by the Life Sciences

Homeworkset 4

Date issued: 4 October 2018.
Date due: 11 October 2018.

1. *Polynomials with positive coefficients*. Consider the system you have selected for Homework Set 1. For polynomial biochemical reaction systems, the appropriate set of polynomials is $\mathbb{R}_+[X_1, \ldots, X_n]$. The properties of the positive real numbers differ from those of the real numbers $\mathbb{R}$.

   Can one add two polynomials of $\mathbb{R}_+[X_1, \ldots, X_n]$ and obtain a product in the same set? Can one multiply two such polynomials in the set? What are the additive and multiplicative identity polynomials? For any polynomial $p \in \mathbb{R}_+[X_1, \ldots, X_n]$ does there exist an additive inverse in the set, thus a polynomial $q \in \mathbb{R}_+[X_1, \ldots, X_n]$ such that $p(x) + q(x) = 0$? Does there exist a multiplicative inverse in the set $\mathbb{R}_+[X_1, \ldots, X_n]$? With which real numbers can one multiply a polynomial in $\mathbb{R}_+[X_1, \ldots, X_n]$ and remain in the same set?

2. *Rational functions*. Consider the set of rational functions denoted by $\mathbb{R}[X_1, \ldots, X_n]$. Which conditions are needed for a rational function in this set to have a multiplicative inverse?

3. *Differential equations of positive rational systems*. Consider the ordinary differential equation of rational form,

$$\begin{aligned} \frac{dx(t)}{dt} &= \frac{c_1 x(t)}{1 + c_2 x(t)} = f(x(t)), \;\; x(0) = 1, \\ & c_1, \; c_2 \in \mathbb{R}, \; c_1 < 0, \; c_2 > 0, X = \mathbb{R}_+. \end{aligned}$$

   Is the rational function of the differential equation nonsingular? What are conditions for the existence and for the uniqueness of a solution of this differential equation? Is the function f bounded over the set $\mathbb{R}_+$? Calculate the limit $\lim_{x\to\infty} \frac{c_1 x}{1+c_2 x}$. Does the limit $\lim_{t\to\infty} x(t; 0, x_0)$ exist and, if so, what value takes the limit?

4. *Rational differential equation*. Consider the rational differential equation,

$$\frac{dx(t)}{dt} = \frac{c_3 x(t)}{(x(t)-1)(x(t)-4)}, \; x(0) = x_0 \in \mathbb{R}.$$

   Discuss the solution of this differential equation for various parts of the real numbers $\mathbb{R}$. Write or display the phase portrait of this system.

Reading advice for Lecture 4

Please read of the lecture notes of Chapter 5 the Sections 5.3, 5.4, 5.5, and 5.6. Note that part of these sections are not yet written.

Reading advice for the future Lecture 5

Lecture 5 will be presented on Tuesday 9 October. Please read of the lecture notes the Sections 6.1, 6.2, and 6.3.