Course Control and System Theory of Rational Systems Motivated by the Life Sciences

Homeworkset 5

Date issued: 9 October 2018.
Date due: 18 October 2018.

1. Prove that the steady state of the following polynomial system is globally asymptotically stable in the indicated state set using the candidate Lyapunov function provided below.

$$\begin{aligned} \frac{dx(t)}{dt} &= -(x(t)-x_s)^3,\ x(0)=x_0, \\ & \quad n_x=1,\ X=\mathbb{R}_+,\ x_s\in\mathbb{R}_{s+}; \\ V(x) &= (x-x_s)^2. \end{aligned}$$

   If you use a theorem of the lecture notes then quote the theorem and argue that the conditions of the theorem are met.

2. Explore the concept of local asymptotic stability. Example 6.4.2 provides a system with this concept. Argue that the steady states $x_{s,1}=1$ and $x_{s,3}=4$ listed in that example are locally asymptotically stable by verification of the conditions.

   Construct an example of a polynomial or rational system of state-space dimension $n_x=2$ or larger, or borrow one from the lecture notes, for which you can prove that a steady state is locally asymptotically stable and not globally asymptotically stable.

Reading advice for Lecture 5

Please read of the lecture notes the Sections 6.1, 6.2, 6.3, and lightly 6.4.

Reading advice for the future Lecture 6

Please read of the lecture notes the Sections 6.5, 6.6, and 6.7. As mentioned before, this is a recommendation only.