Course Control and System Theory of Rational Systems Motivated by the Life Sciences

Homeworkset 7

Date issued: 18 October 2018.
Date due: 25 October 2018.

1. *Controllability*. Please check the controllability of the following structured linear system which system representation is slightly different from that considered in Chapter 7.

$$
\begin{aligned}
\frac{dx(t)}{dt} &= Ax(t) + Bu(t),\ x(0) = x_0, \\
A &= \begin{pmatrix} a_{11} & 1 & 0 & 1 \\ a_{21} & 0 & 1 & 1 \\ a_{31} & 0 & 0 & 1 \\ 0 & 0 & 0 & a_{44} \end{pmatrix},\ B = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \\
& n_x = 4,\ n_u = 1,\ a_{11},\ a_{21},\ a_{31},\ a_{44} \in \mathbb{R}\backslash\{0\}.
\end{aligned}
$$

2. *Observability*. Consider a time-invariant linear systems. Call the observability map *injective* if the next displayed map is injective,

$$x_0 \mapsto y(*; 0, x_0) = \{y(t; 0, x_0) \in Y,\ \forall\, t \in [0, \infty)\}.$$

Recall that a map $h : X \to Y$ is called injective if $\forall\, x_1, x_2 \in X$, $h(x_1) = h(x_2)$ implies that $x_1 = x_2$. Prove equivalence of the following two statements:

(a) The linear system is observable according to the characterization of Th. 7.3.11 of the lecture notes.

(b) The observability map of the system is injective as defined above.

3. *Observable canonical form.* Consider the specific form of a time-invariant linear system of state-space dimension 3 and output dimension 2.

$$
\begin{aligned}
\frac{dx(t)}{dt} &= Ax(t),\ x(0) = x_0, \\
y(t) &= Cx(t), \\
A &= \begin{pmatrix} 0 & 1 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix},\ C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\
&= n_x = 3,\ n_y = 2,\ a_{21},\ a_{22},\ a_{31},\ a_{32},\ a_{33} \in \mathbb{R}.
\end{aligned}
$$

Prove by the following three steps that this particular form is a canonical form.

(a) Prove that any system of this form is observable.

(b) If there are two systems of the above form possibly with different values for the parameters $a_{i,j}$ which are similar as defined in Def. 7.4.1 of the lecture notes, then the system matrices A and C of the considered two systems have to be identical.

(c) Prove that any linear system with observability indices equal to $n_{obs} = (2, 1)$ can be transformed to the above defined special form.

It follows from the three properties that the above defined special form is an observable canonical form.

Reading advice for Lecture 7

Please read of the lecture notes the Sections 7.7 - 7.6. The reading of the proof of Section 7.5 is not urgent.

Reading advice for the future Lecture 8

On Thursday 23 October, Lecture 8 will be presented. Please read of the lecture notes the Sections 7.7 - 7.12. Several of these sections are not yet entered in the lecture notes. As mentioned before, this advice is a recommendation only.