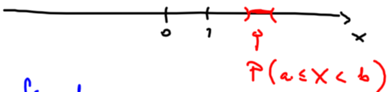
X



$$EX = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$\boxed{\operatorname{var} X = E(X-EX)^2}$$

$$\operatorname{cov}(X,Y) = E(X-EX)(Y-EY)$$

Theorem: X, Y nezávislé

$$\Rightarrow E(XY) = (EX)(EY)$$

12 6-15:57

(X,Y) $F_{X,Y}$, F_X, F_Y

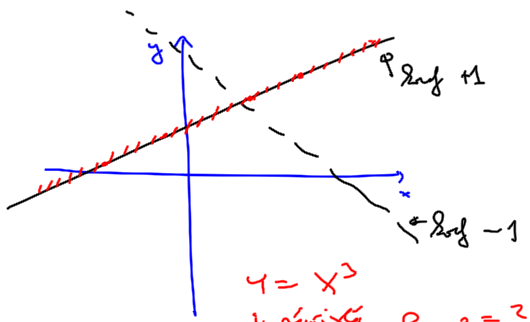
$X<x$, $Y<y$ jsou nezávislé jevy

$$\Leftrightarrow F_{X,Y}(x,y) = F_X(x)\cdot F_Y(y)$$

$$\Leftrightarrow \begin{cases} f(x,y) = f_X(x)\cdot f_Y(y) & \text{pro pravd. fce} \\ f(x,y) = f_X(x)\cdot f_Y(y) & \text{pro hustoty} \end{cases}$$

$$E(X\cdot Y) = \sum_{i,j} x_i y_j\, f_X(x_i)\, f_Y(y_j) = \Big(\sum x_i f_X(x_i)\Big)(\cdot/.)$$

12 6-16:11



$Y = X^3$

p závislé, $\rho_{X,X^3} = ?$

$\rho_{X,Y} \sim 1$

12 6-16:22

$$\mu'_k = EX^k = \int_{-\infty}^{\infty} x^k f_X(x)\, dx$$

$$M_X(t) = \sum_{k=0}^{\infty} EX^k \frac{t^k}{k!} = E e^{tX}$$

$$E e^{t(X+Y)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{t(x+y)} f_{X,Y}(x,y)\, dx\, dy$$

$$M_{X+Y}(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{tx} f_X(x)\, e^{ty} f_Y(y)\, dx\, dy$$

$$= \Big(\int_{-\infty}^{\infty} e^{tx} f_X(x)\, dx\Big)\Big(\int_{-\infty}^{\infty} e^{ty} f_Y(y)\, dy\Big) = M_X(t)\, M_Y(t)$$

12 6-16:29

$$\frac{d}{dt}\Big|_{t=0} \big(p(e^t-1)+1\big)^n = n(\cdot/.)^{n-1} p e^t \Big|_{t=0} = np$$

$$\operatorname{var} X = EX^2 - (EX)^2 = \cancel{n^2 p^2} - np^2 + np - \cancel{n^2 p^2} = np(1-p)$$

2. derivace

$$\frac{d^2}{dt^2}\Big|_{t=0} \cdot/. = n(n-1)(\cdot/.)^{n-2} p^2 e^{2t} + n(\cdot/.)^{n-1} p e^t \Big|_0$$

$$= n\cdot(n-1)p^2 + np$$

$$Z \rightsquigarrow \boxed{\sum_{k=0}^{\infty} \frac{1}{k!}\frac{1}{2^k} t^{2k}}$$

12 6-16:42

$$X \rightsquigarrow Y = \frac{1}{\sqrt{\operatorname{var} X}}(X - EX)$$

$$E e^{tX} = \int_{-\infty}^{\infty} e^{tx} f_X(x)\, dx$$

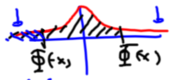
$Y = a + bX$

$$E e^{tY} = \int_{-\infty}^{\infty} e^{at} e^{btx} f_X(x)\, dx = M_X(bt)\cdot e^{at}$$

$$e^x = \lim_{n\to\infty}\Big(1+\frac{x}{n}\Big)^n$$

12 6-16:52

$X \sim Bi(n,p)$



$\boxed{P\left(\left|\frac{1}{n}X - p\right| < 0{,}05\right) \geq 0{,}9}$ pro velká n je $=$

$0{,}9 \simeq P\left(\left|\frac{1}{n}X - p\right| < 0{,}05\right) =$

$= P\left(\frac{-0{,}05n}{\sqrt{np(1-p)}} < \frac{X-np}{\sqrt{np(1-p)}} < \frac{0{,}05n}{\sqrt{np(1-p)}}\right)$

$= \Phi\left(\frac{0{,}05n}{\sqrt{np(1-p)}}\right) - \Phi\left(\frac{-0{,}05n}{\sqrt{np(1-p)}}\right)$

$= 2\Phi\left(\frac{0{,}05n}{\sqrt{np(1-p)}}\right) - 1 \Rightarrow \Phi\left(\frac{0{,}05n}{\sqrt{np(1-p)}}\right) \approx \frac{1}{2}(1+0{,}9)$
$= 0{,}95$
$= z(0{,}05)$

12 6-17:06

$X_i \simeq N(\mu_i, \sigma_i^2)$ $\quad X_i = \mu_i + \sigma_i Z$

$M_{X_i}(t) = e^{\mu_i t} e^{\sigma_i^2 t^2/2}$

$M_{X_1+\dots+X_n}(t) = \prod_i e^{\mu_i t} e^{\sigma_i^2 t^2/2}$

$= e^{(\mu_1+\dots+\mu_n)t} e^{(\sigma_1+\dots+\sigma_n)t/2}$

$X_1 + \dots + X_n \sim N(\mu_1 + \dots + \mu_n, (\sigma_1 + \dots + \sigma_n)^2)$

$\frac{1}{n}(X_1 + \dots + X_n)$

12 6-17:28