$f : V \longrightarrow W$ lineární zobrazení

$$\mathbb{R}^n \longrightarrow \mathbb{R}^m$$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \xmapsto{f} \begin{pmatrix} \sum_{j=1}^{n} a_{ij} x_j \end{pmatrix}$$

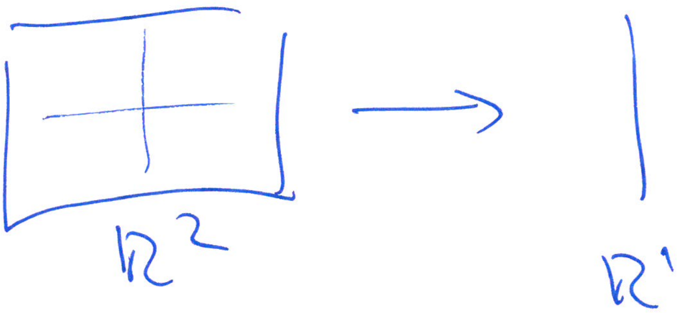
i-tý člen ve výsledku

$$x \longmapsto A \cdot x$$

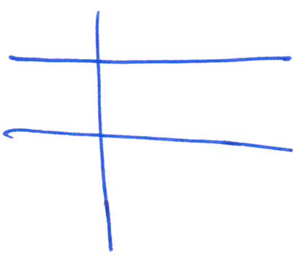
speciální případ: lineární funkce

$m = 1$

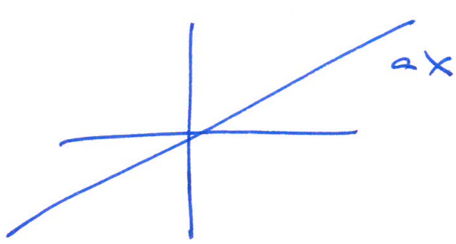
$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \longmapsto f(x_1, \dots, x_n) = \underbrace{a_1 x_1}_{} + a_2 x_2 + \dots + a_n x_n$$



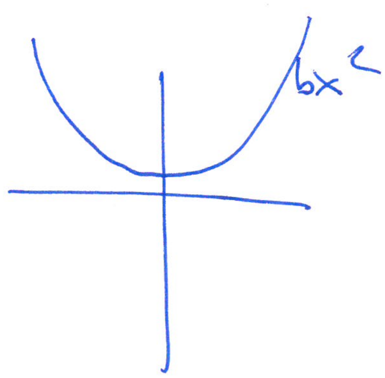
$$(x, y) \longmapsto 2x - y$$



konstanta



lineární



kvadratické

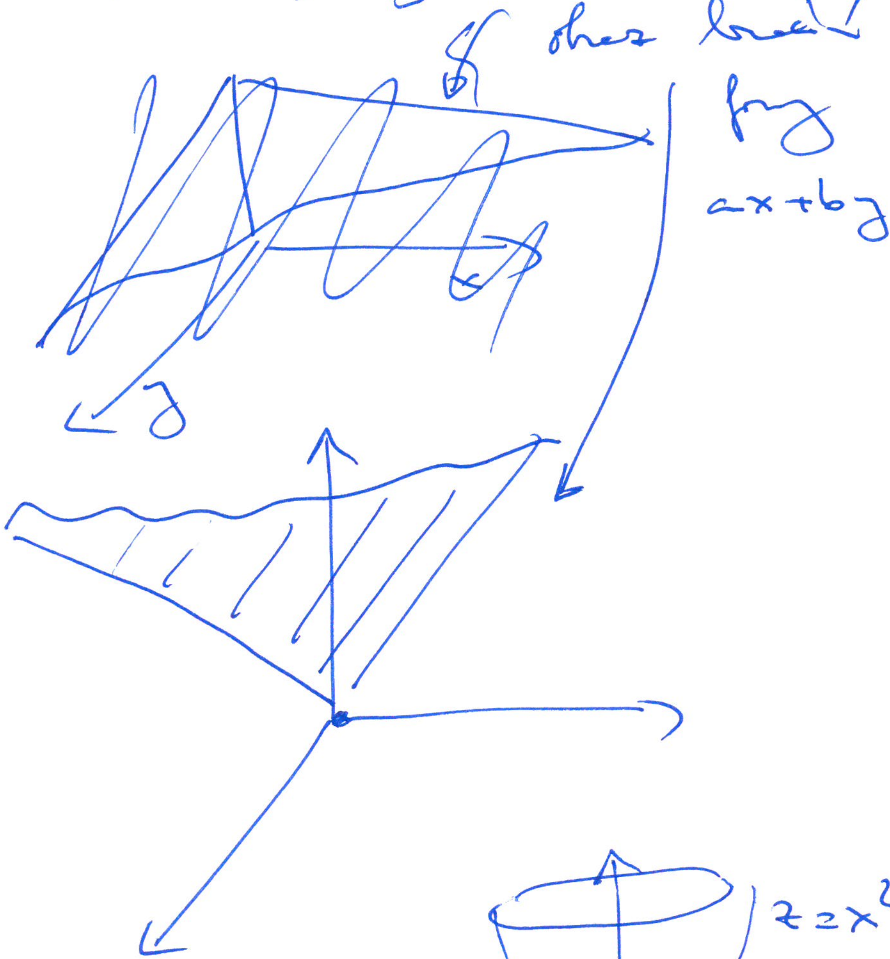
na $\mathbb{R}^n$ kvadratické formy:

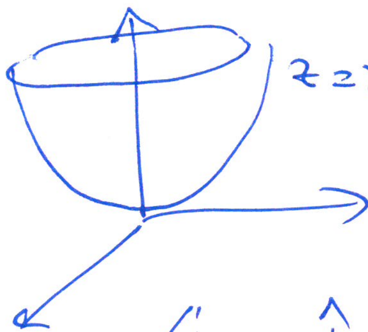
$n=2$ $\quad \boxed{ax^2+2bxy+cy^2}$

lineární: $\quad (a\ \ b)\cdot\begin{pmatrix}x\\y\end{pmatrix} = ax+by$

kvadratické: $\quad (x\ \ y)\cdot\begin{pmatrix}a & b\\ b & c\end{pmatrix}\cdot\begin{pmatrix}x\\y\end{pmatrix} = ax^2+2bxy+cy^2$

lineární

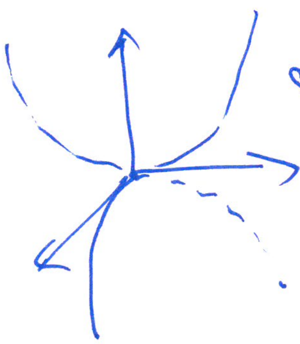




$z=x^2+y^2$

1) x^2+y^2

kvadratické



sedlo

2) x^2-y^2

3) x^2

Definiční obor funkce :

$$f(x,y) = \frac{x+y}{x^2+y^2} \qquad D = \{(x,y);\ x\neq 0,\ y\neq 0\}$$

parciální derivace :

$$\frac{\partial}{\partial x} f(x,y) = \frac{d}{dx} f(x,\underbrace{y}_{\text{(konstanta)}})$$

$$= \frac{1\cdot(x^2+y^2) - (x+y)\,2x}{(x^2+y^2)^2} = \frac{-x^2+y^2-2xy}{(x^2+y^2)^2}$$

$$\frac{\partial}{\partial y} f(x,y) = \frac{x^2-y^2-2xy}{(x^2+y^2)^2}$$

v bodě $(1,1)$ $\qquad \frac{\partial}{\partial x} f(1,1) = -\frac{1}{2}$

$$\frac{\partial}{\partial y} f(1,1) = -\frac{1}{2}$$

lineární forma přibližující $f(x,y)$ v $(\overset{x_0}{1},\overset{y_0}{1})$

$$\boxed{D^1 f(x,y) = f(1,1) - \frac{1}{2}(x-1) - \frac{1}{2}(y-1)}$$

$$\mathbb{R}^n = \left\langle \begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix}, \begin{pmatrix}0\\1\\\vdots\\0\end{pmatrix}, \dots, \begin{pmatrix}0\\0\\\vdots\\1\end{pmatrix} \right\rangle$$

ortonormální báze

$$\langle x, y \rangle = x^T \cdot y = (\,\cdots\,) \cdot \begin{pmatrix}\vdots\end{pmatrix}$$

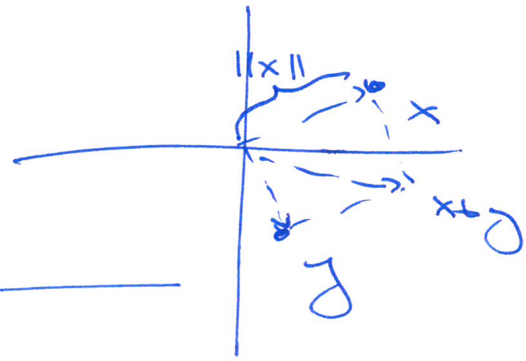
$$\|x\|^2 = \langle x, x \rangle = x_1^2 + x_2^2 + \dots + x_n^2$$

1) $\|x\| \geq 0$, $\|x\| = 0 \iff x = 0$

2) $\|ax\| = |a|\,\|x\|$

konstanta — abs. hodnota

3) $\|x+y\| \leq \|x\| + \|y\|$



vzdálenost

$$d(x,y) = \|x-y\|$$

1) $d(x,y) \geq 0$, $d(x,y) = 0 \iff x = y$

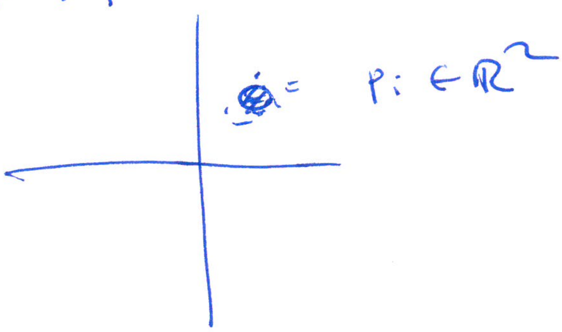
2) $d(x,y) = d(y,x)$

3) $d(x,z) \leq d(x,y) + d(y,z)$

Metrický prostor (M,d)

$d: M \times M \to \mathbb{R}$

$p_1, p_2, \dots$ posloupnost v M



Cauchyovská:

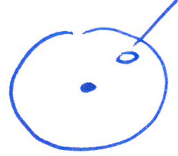
konvergentní: $\lim_{n\to\infty} p_n = p$

v $\mathbb{R}^n$: $d(x,y) = \sqrt{(x_1-y_1)^2 + \dots + (x_n-y_n)^2}$

body $p^1, p^2, \dots$, posloupnost konverguje k p:

$\boxed{\forall \varepsilon\ \exists N\ p_n \in O_\varepsilon(p)\ \forall n \geq N.}$

$O_\varepsilon(p)$



$O_\varepsilon(p) = \{x ; d(x,p) \leq \varepsilon\}$

Podmnožiny v $\mathbb{R}^n$

1) otevřené U: $\forall p \in U\ \exists O_\varepsilon(p) \subset U$

2) uzavřené K: $\forall p_1, p_2, \dots \in K$ $\lim_{i \to \infty} p_i = p \Rightarrow p \in K$

platí $\mathbb{R}^n \setminus U$ je uzavřené, $\mathbb{R}^n \setminus K$ otevřené.

hromadný bod množiny U = limita nějaké posloupnosti bodů v U, různých od ní!

$f: \mathbb{R}^n \to \mathbb{R}$ definovaná na U, p hromadný bod