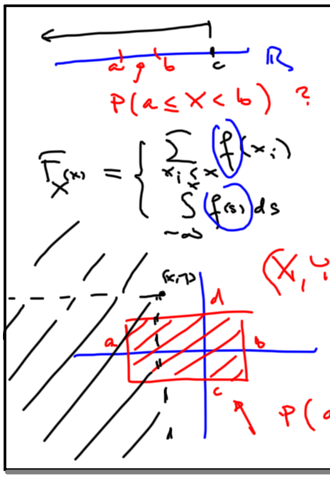

$P(a \le X < b)$?

X

$F_X(x) = P(X < x)$

$F_X(x) = \begin{cases} \sum_{x_i < x} f(x_i) & X \text{ diskrétní} \\ \int_{-\infty}^{x} f(s)\,ds & \text{spojité} \end{cases}$

(X, Y)

$F_{X,Y}(x,y) = P(X<x, Y<y)$

$\Downarrow$

$F_X(x) = F_{X,Y}(x, \infty)$

$P(a \le X \le b,\ c \le Y \le d)$

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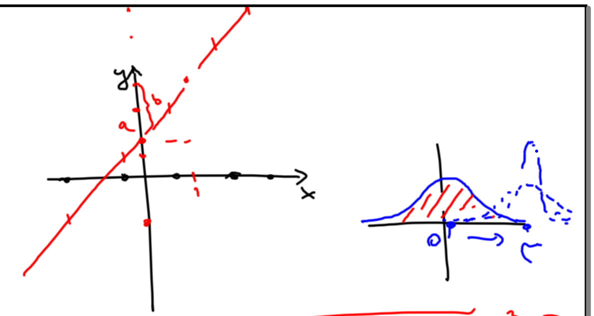

$N(0,1) \sim Z$ .. hustota $\boxed{\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}}$

$Y = \mu + \sigma Z$

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$V = X + Y$, X, Y nezávislé

$F_V(u) = \iint_{x+y<u} f_X(x) f_Y(y)\,dx\,dy$

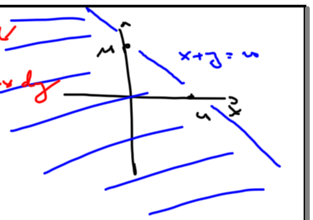
$= \int_{-\infty}^{\infty} \int_{-\infty}^{u-x} f_Y(y) f_X(x)\,dy\,dx$

$\left\{ \begin{matrix} v = x+y \\ dv = dy \end{matrix} \right\}$

$= \int_{-\infty}^{u} \left(\int_{-\infty}^{\infty} f_X(x) f_Y(v-x)\,dv \right) dx$

$\qquad \parallel$

$(f_X * f_Y)(x)$



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$X \sim f(2^k) = 2^{-k}$

EX ? neexistuje: $\sum_{k=1}^{\infty} 1 = \sum 2^k \cdot 2^{-k}$

$X \sim A(p)$

$EX = 0(1-p) + 1 \cdot p = p$

$Y = X_1 + \dots + X_n$, $X_i \sim A(p)$ nezávislé

$EY = \sum_{k=0}^{n} k \binom{n}{k} p^k (1-p)^{n-k} \stackrel{?}{=} np$

lis 20-14:41

$\boxed{Z = N(0,1)}$

$EZ = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z e^{-\frac{z^2}{2}}\,dz = 0$

$\boxed{Y = \mu + \sigma Z}$ $\quad EY$? $\quad z < \frac{1}{\sigma}(y-\mu)$

$F_Y(y) = P(Y<y) = P(\mu + \sigma Z < y)$

$= F_Z\left(\frac{1}{\sigma}(y-\mu)\right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{(y-\mu)/\sigma} e^{-z^2/2}\,dz \quad \left\{ \begin{matrix} x = \mu + \sigma z \\ dx = \sigma\,dz \end{matrix} \right\}$

$= \boxed{\frac{1}{\sigma\sqrt{2\pi}}} \int_{-\infty}^{y} \boxed{e^{-(x-\mu)^2/2\sigma^2}}\,dx$

$EY = \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx \left\{ \begin{matrix} x - \mu = y \\ dx = dy \end{matrix} \right\} = \mu$

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Součet $EX + EY$ po spojité:

$E(X+Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x+y) f_{X,Y}(x,y)\,dx\,dy$

$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x f_{X,Y}(x,y)\,dx\,dy + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y f_{X,Y}(x,y)\,dx\,dy$

$= \int_{-\infty}^{\infty} x f_X(x)\,dx + \int_{-\infty}^{\infty} y f_Y(y)\,dy = EX + EY$

Vektorově: PAJŠI! $W = (X_1, \dots, X_n)^T$

$B \in Mat_{mn}$

$\boxed{E(a + BW) = a + B \cdot EW}$ $\quad a \in \mathbb{R}^m$

$EW = (EX_1, \dots, EX_n)^T$

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$$\begin{array}{cc} & n \\ m & \left(B \right) \end{array} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} a_1 \\ \vdots \\ a_m \end{pmatrix}$$

$B \quad W$

$$\operatorname{var} X = \boxed{E(X-EX)^2} = E(X^2 - 2X\,EX + (EX)^2)$$
$$= EX^2 - 2(EX)^2 + (EX)^2 = EX^2 - (EX)^2$$

X, Y nezávislé

$$\operatorname{var} XY = E(X\cdot Y)^2 - (E(XY))^2 = EX^2 \cdot EY^2 - (EX)^2(EY)^2$$

$\parallel (?)$

$\operatorname{var} X \operatorname{var} Y$

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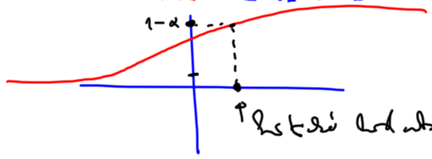
$$Z = N(0,1) \qquad EZ = 0$$

$-z e^{-z^2/2}$

$$\operatorname{var} Z = EZ^2 - 0^2 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2 e^{-z^2/2}\, dz = \frac{2}{\sqrt{2\pi}} \int_0^{\infty} z^2 e^{-z^2/2}\, dz$$

$$\begin{cases} z^2/2 = x \\ z\, dz = dx \end{cases} \qquad = \frac{2}{\sqrt{2\pi}} \int_0^{\infty} \sqrt{2x}\, e^{-x}\, dx$$

$$1 - \quad = \ldots = 1$$



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