$$ax^2+2bxy+cy^2$$
$$\updownarrow$$
$$d_1x^2+d_2y^2$$
$\neq 0$ $\neq 0$

říj 2-14:01

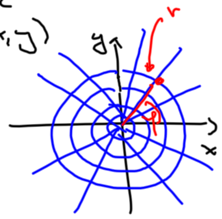
$$F:\mathbb{R}^n\to\mathbb{R}^m$$
$$\begin{pmatrix}x_1\\ \vdots\\ x_n\end{pmatrix}\xmapsto{F}\begin{pmatrix}f_1(x_1,\dots,x_n)\\ \vdots\\ f_m(x_1,\dots,x_n)\end{pmatrix}$$

Příklad: $\mathbb{R}^2\to\mathbb{R}^2$

(r,φ) (x,y)

$x=r\cos\varphi$

$y=r\sin\varphi$



$\mathbb{R}^2=\mathbb{C}\to\mathbb{C}$

$z=x+iy\longmapsto z^2=(x^2-y^2)+i\,2xy$

říj 2-14:09

Speciálně:
$$\begin{pmatrix}x_1\\ \vdots\\ x_n\end{pmatrix}\longmapsto\begin{pmatrix}a_{11}\cdots a_{1n}\\ \vdots\\ a_{m1}\cdots a_{mn}\end{pmatrix}\begin{pmatrix}x_1\\ \vdots\\ x_n\end{pmatrix}=\begin{pmatrix}y_1\\ \vdots\\ y_m\end{pmatrix}$$
LINEÁRNÍ

Diferenciál: $F:\mathbb{R}^n\to\mathbb{R}^m$ $F=(f_1,\dots,f_n)$
$$D^1F=\begin{pmatrix}df_1\\ \vdots\\ df_m\end{pmatrix}=\begin{pmatrix}\frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n}\\ \vdots & & \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n}\end{pmatrix}$$
$\Rightarrow$ JACOBIHO MATICE $(m\times n)$

Speciální případy: $n=1$ .. křivka, $m=1$ .. funkce

říj 2-14:14

$$F(x+v)-F(x)=\boxed{D^1F(x)(v)}+\alpha(v)$$
bod v $\mathbb{R}^n$ $\mathbb{R}^n$ přírůstek
$$\boxed{\lim_{v\to 0}\frac{1}{\|v\|}\alpha(v)=0}$$

Příklad: $x=r\cos\varphi$, $y=r\sin\varphi$ $\Big\}$ $F(r,\varphi)=\begin{pmatrix}x(r,\varphi)\\ y(r,\varphi)\end{pmatrix}$
$$D^1F=\begin{pmatrix}\cos\varphi & -r\sin\varphi\\ \sin\varphi & r\cos\varphi\end{pmatrix}$$

říj 2-14:19

$$\underbrace{\mathbb{R}^n\xrightarrow{F}\mathbb{R}^m\xrightarrow{G}\mathbb{R}^r}_{G\circ F}$$
$$\begin{pmatrix}g_1\circ F\\ g_2\circ F\\ \vdots\\ g_r\circ F\end{pmatrix}=G\circ F$$

Diferenciály:
$$\underbrace{x\in\mathbb{R}^n\xrightarrow{D^1F(x)}\underset{F(x)}{\mathbb{R}^m}\xrightarrow{D^1G(F(x))}\mathbb{R}^r}_{D^1(G\circ F)(x)}$$

pro $n=m=r=1$: $(g\circ f)'(t)=g'(f(t))\cdot f'(t)$

říj 2-14:30

Dk. Umíme pro $n=1$, $r=1$:
$$\underbrace{\mathbb{R}\xrightarrow{c}\mathbb{R}^m\xrightarrow{f}\mathbb{R}}\qquad (f\circ c)'=f_{x_1}c_1'+\dots+f_{x_m}c_m'$$
$$=(f_{x_1}\cdots f_{x_m})\cdot\begin{pmatrix}c_1'\\ \vdots\\ c_m'\end{pmatrix}$$

Stačí pro $r=1$:
$$d_v(g\circ F)(x)=\lim_{t\to 0}\frac{1}{t}\left(g\left(F(x+tv)\right)-g\left(F(x)\right)\right)$$
dosadíme lineární aprox.
$$\boxed{d_v(g\circ F)(x)=dg\circ D^1F(x)(v)}$$

říj 2-14:36

Příklad $\quad (r,\varphi) \quad (x,y)$

$$\mathbb{R}^2 \underset{F^{-1}}{\overset{F}{\rightleftarrows}} \mathbb{R}^2 \qquad \left.\begin{matrix} x = r\cos\varphi \\ y = r\sin\varphi \end{matrix}\right\} = F$$

$$\boxed{g(r,\varphi,t) = \sin(r-t)} \qquad r = \sqrt{x^2+y^2}, \quad \varphi = \operatorname{arctg}\frac{y}{x}$$

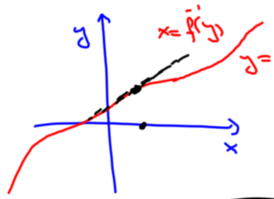
$$\frac{\partial g}{\partial x} = \frac{\partial g}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial g}{\partial \varphi}\cdot\frac{\partial \varphi}{\partial x}$$

$$\frac{\partial g}{\partial y} = \frac{\partial g}{\partial r}\cdot\frac{\partial r}{\partial y} + \frac{\partial g}{\partial \varphi}\cdot\frac{\partial \varphi}{\partial y}$$

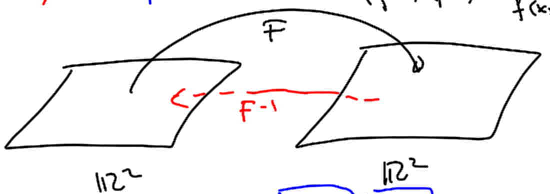
$$\to = \cos(r-t)\cdot\frac{1}{\cancel{2}}\frac{\cancel{2}x}{\sqrt{x^2+y^2}} + 0 = \cos\left(\sqrt{x^2+y^2}-t\right)\cdot\frac{x}{\sqrt{x^2+y^2}}$$

$$\to = \cos\left(\sqrt{x^2+y^2}-t\right)\cdot\frac{y}{\sqrt{x^2+y^2}}$$

říj 2-14:45



$$f'(x) \neq 0$$
$$\Downarrow$$
f^{-1} existuje na okolí
$$(f^{-1})'(f(x)) = \frac{1}{f'(x)}$$

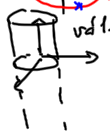


$$\boxed{E} = D^1(F^{-1}\circ F) = \boxed{D^1F^{-1}}\cdot\boxed{D^1F}$$

říj 2-14:59

$F(x,y)$: $x^2+y^2-r^2=0$ v $\mathbb{R}^2$:

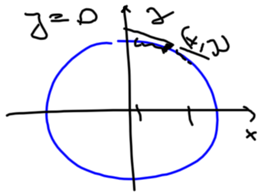
$F(x,y,z)$ v $\mathbb{R}^3$:



IMPLICITNÍ ZADÁNÍ

kdy lze zadat $y = y(x)$

$$\frac{\partial F}{\partial y} = 2y = 0 \quad \text{pro } y = 0$$



říj 2-15:11

předp. $F(x,y) = 0$, $F_y(x,y) \neq 0$

$$\boxed{F(x,y(x)) = 0} \qquad y = y(x)$$

$$0 = \frac{d}{dx}F(x,y(x)) = \underline{F_x(x,y(x)) + F_y(x,y(x))\,y'(x)}$$

$$\Rightarrow \quad y'(x) = -\frac{F_x(x,y(x))}{F_y(x,y(x))}$$

pro kružnici $F(x,y) = x^2+y^2-r^2$

$$y'(x) = -\frac{2x}{2y} = -\frac{x}{y}$$

říj 2-15:21

$$D^1F = (F_x \;\; F_y) \qquad F(\underset{\mathbb{R}^m}{x}, \underset{\mathbb{R}^n}{y}) \in \mathbb{R}^n$$

$$D^1F = \left.\Big(\underbrace{D^1_xF}_{m} \;\Big|\; \underbrace{\boxed{D^1_yF}}_{n}\Big)\right\}n$$

invertibilní

$G: \mathbb{R}^m \to \mathbb{R}^n$ tak, že:

$$\boxed{F(x,G(x)) \equiv 0}$$

jednoznačně definované

$$D^1G = -(D^1_yF)^{-1}\cdot D^1_xF$$

$$\tilde{F}(x,y) = (x, F(x,y)) \qquad \begin{pmatrix} E & 0 \\ D_xF & D_yF \end{pmatrix}$$

říj 2-15:31

říj 2-15:37