$\bar X = \frac{1}{n}\sum_{i=1}^{n} X_i$

$E\bar X = \frac{1}{n}\sum_{i=1}^{n} EX_i = \frac{1}{n} n\mu = \mu$

$var\,\bar X = var\,\frac{1}{n}\sum X_i = \frac{1}{n^2}\sum_i var\,X_i = \frac{1}{n^2}\cdot n\sigma^2$

$X_i \sim N(\mu, \sigma^2)$

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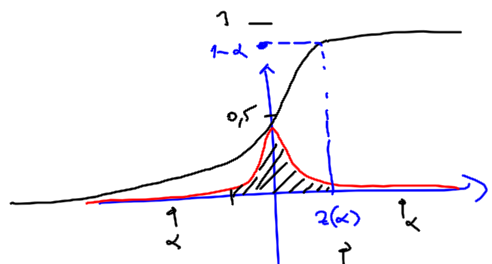

$\bar X > \mu$:

$\bar X - \mu < \sqrt{\sigma^2/n}\cdot z(\alpha/2)$

$\bar X - \frac{\sigma}{\sqrt{n}} z(\alpha/2) < \mu$

$\alpha \mapsto \alpha/2$

$\bar X < \mu$:

$\mu - \bar X < \sigma/\sqrt{n}\cdot z(\alpha/2)$

$\mu < \bar X + \frac{\sigma}{\sqrt{n}} z(\alpha/2)$

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$Z = \frac{\bar X - \mu}{\sigma}\sqrt{n}$ $\quad T = \frac{\bar X - \mu}{S}\sqrt{n}$

Φ

t-rozděl.

$\frac{1}{n}\sum(X_i - \bar X)^2 = \frac{n-1}{n}\cdot S^2$

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$(X_1, \dots, X_n)$

$f(x_1, \dots, x_n, \theta) = f_X(x_1, \theta)\cdots f_X(x_n, \theta)$

statistika T ... $\hat\theta$ je, který je lokální max.

$X_i \sim N(\underbrace{\mu, \sigma^2}_{\theta})$ $\quad f_X(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$l(x_1, \dots, x_n, \mu, \sigma^2) = \ln\left(\prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}\right)$

$= -n\frac{1}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$

max funkce l: $\boxed{\frac{\partial l}{\partial \mu} = 0}$ $\quad \frac{\partial l}{\partial \sigma^2} = 0$

$\frac{\partial l}{\partial \mu}$: $\frac{\cancel{2}}{\cancel{2}\sigma^2}\sum_{i=1}^{n}(x_i - \mu) = 0 \Rightarrow \sum_{i=1}^{n} x_i = n\mu$

$\frac{\partial l}{\partial \sigma^2}$: ---> $\hat\sigma^2 = S^2$ ✓

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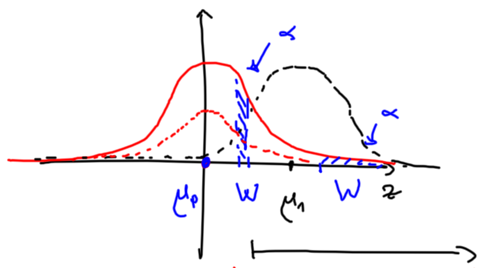

c=1:

c=1/2:

$W_c = \{ f(x, \theta_1) \ge c\, f(x, \theta_0) \}$ $\quad P(W_c)$

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