$$f(x+v) = f(x) + D^1 f(x)(v) + \tfrac{1}{2} D^2 f(x+\theta\cdot v)(v)$$

$x \in \mathbb{R}^n$, $v \in \mathbb{R}^n$

$$D^1 f(x)(v) = (f_{x_1} \cdots f_{x_n}) \cdot \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}, \qquad D^2 f(x+\theta\cdot v)(v) = (v_1 \cdots v_n) \cdot H \cdot \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}, \qquad H = \begin{pmatrix} f_{x_1x_1} & \cdots & f_{x_1x_n} \\ \vdots & & \vdots \\ f_{x_nx_1} & \cdots & f_{x_nx_n} \end{pmatrix}$$

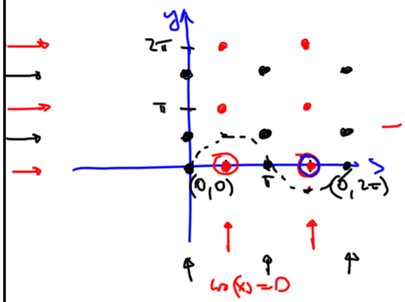
$n=2$ (x,y)

$$Hf(x,y) = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}\Bigg|_{(x,y)}$$

10 4-16:03

$$f(x,y) = \sin(x)\cos(y)$$

② $\cos y = 0$, $\sin x = 0$



$$Hf = -\begin{pmatrix} \sin x \cos y & \cos x \sin y \\ \cos x \sin y & \sin x \cos y \end{pmatrix}$$

$$H = \mp\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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$$(u\ v)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = (v\ u)\cdot\begin{pmatrix} u \\ v \end{pmatrix} = 2uv$$

$$\boxed{2x^2 - 4xy + 2y^2 + zy} =$$

$$\begin{pmatrix} 2 & -2 & 0 \\ -2 & 2 & 1/2 \\ 0 & 1/2 & 0 \end{pmatrix} \quad 2,\ 0,\ -1/2$$

$$= 2\underbrace{(x-y)^2}_{= x^2-2xy+y^2} + y^2 + zy = 2(x-y)^2 + \left(y + \tfrac{1}{2}z\right)^2 - \tfrac{1}{4}z^2$$

$x \in \mathbb{R}^n$ symetrická matice S typu $n\times n$

$$x \mapsto x^T S x$$

$$Px \mapsto x^T \underbrace{\boxed{P^T S P}}_{P^T = P^{-1}} x$$

$$S \rightsquigarrow \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix}$$

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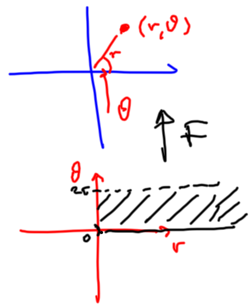
$\mathbb{R}^2 \to \mathbb{R}^2$

$(r,\theta) \quad (x,y)$

$$x = r\cos\theta = f(r,\theta)$$
$$y = r\sin\theta = g(r,\theta)$$

$$DF = \begin{pmatrix} f_r & f_\theta \\ g_r & g_\theta \end{pmatrix} = \begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix}$$



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$$\underset{t}{\mathbb{R}} \xrightarrow{f} \underset{x}{\mathbb{R}} \xrightarrow{g} \underset{y}{\mathbb{R}}$$

$$y = g(x) = g(f(t))$$

$$\frac{dy}{dt} = \frac{dy}{dx}\,\frac{dx}{dt}$$

$$\frac{d}{dt} y(t) = \boxed{g'(f(t))\cdot f'(t)}$$

$$\mathbb{R}^n \xrightarrow{F} \mathbb{R}^m \xrightarrow{G} \mathbb{R}^r, \qquad G\circ F$$

$$\mathbb{R}^n \xrightarrow{D^1F} \mathbb{R}^m \xrightarrow{D^1G} \mathbb{R}^r, \qquad D^1G \cdot D^1F$$

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$$\mathbb{R}^2 \xrightarrow{F} \mathbb{R}^2 \xrightarrow{g} \mathbb{R}$$

$(x,y) \quad (r,\varphi)$

$$\left(\boxed{\frac{\partial(g\circ F)}{\partial x}}\ \boxed{\frac{\partial(g\circ F)}{\partial y}}\right) = \underbrace{\left(\frac{\partial g}{\partial r}\ \ \frac{\partial g}{\partial \varphi}\right)}_{D^1 g}\cdot\begin{pmatrix} \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} \\ \frac{\partial \varphi}{\partial x} & \frac{\partial \varphi}{\partial y} \end{pmatrix}$$

$$\left(\frac{\partial g}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial g}{\partial \varphi}\cdot\frac{\partial \varphi}{\partial x} \qquad \frac{\partial g}{\partial r}\cdot\frac{\partial r}{\partial y} + \frac{\partial g}{\partial \varphi}\frac{\partial \varphi}{\partial y}\right)$$

$$g(r,\varphi) = \sin(r-t)$$

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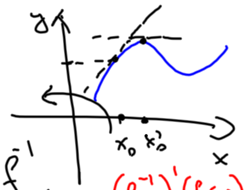
$\mathbb{R} \underset{f^{-1}}{\overset{f}{\rightleftarrows}} \mathbb{R}$?



$f'(x_0) \neq 0 \Rightarrow$ ex f^{-1} na okolí

$(f^{-1})'(f(x_0)) = \frac{1}{f'(x_0)}$

$\mathbb{R}^n \xrightarrow{F} \mathbb{R}^n$

$D^1F(x_0)$

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Předp. že pro $y = F(x)$ ex. diferenc. $x = F^{-1}(y)$ v okolí x_0 a $y_0 = F(x_0)$

$\Rightarrow D^1(F^{-1} \circ F) = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} = (D^1F^{-1}) \circ (D^1F)$

inverzní matice

$\begin{pmatrix} \cos\varphi & -r\sin\varphi \\ \sin\varphi & r\cos\varphi \end{pmatrix} =: D^1F$

$\boxed{\frac{1}{r}\begin{pmatrix} r\cos\varphi & r\sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}}$

$\boxed{r = \det(D^1F)}$

$x = r\cos\varphi$ | $r = \sqrt{x^2+y^2}$

$y = r\sin\varphi$ | $\varphi = \operatorname{arctg}\frac{y}{x}$

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