IA168 — Problem set 3

For an extensive-form game G, let SPE(G) denote the set of subgame-perfect equilibria of G and NE(G) denote the set of Nash equilibria of G.

Problem 1 [6 points]

Consider the following two-player strategic-form game G:

	C	S
C	$(-5,-5)$	$(0,-20)$
S	$(-20,0)$	$(-1,-1)$

a) Calculate the number of strategies of Player 1 and Player 2 in G_{2-rep};

b) calculate the number of strategy profiles in G_{2-rep};

c) calculate the number of Nash equilibria in G_{2-rep}.

Moreover, in dependence on parameter $t \in \mathbb{Z}^+$

d) calculate the number of strategies of Player 1 and Player 2 in G_{t-rep}.

e) calculate the number of strategy profiles in G_{t-rep}.

f) find all subgame perfect equilibria in G_{t-rep}.

Use the definition, not the example from the lecture. Justify your reasoning.

Problem 2 [5 points] Consider the following two-player strategic-form game G

	X	Y
A	$(4,4)$	$(-1,5)$
B	$(5,-1)$	$(1,1)$

a) In G^{avg}_{irep}, find a subgame-perfect equilibrium whose outcome is $(3.2, 3.5)$.

b) Calculate $\inf_{s \in \mathrm{SPE}(G^{avg}_{irep})} u_1(s)$.

c) Calculate $\sup_{s \in \mathrm{SPE}(G^{avg}_{irep})} u_1(s)$.

Justify your reasoning.

Problem 3 [4 points] Give an example of a two-player strategic-form game $G = (\{1,2\}, (S_1, S_2), (u_1, u_2))$ such that all of the following conditions are satisfied

a) $|S_1| + |S_2| = 5$;

b) $\max_{s \in \mathrm{SPE}(G^{avg}_{irep})} u_1(s) = 0$;

c) $\max_{s \in \mathrm{NE}(G^{avg}_{irep})} u_1(s) = 5$.

Find the SPE s such that $u_1(s) = 0$ and NE s' such that $u_1(s') = 5$. Explain your reasoning.

Problem 4 [5 points] Consider the following strategic-form game G

	A_2	B_2
A_1	$(2,1)$	$(7,-1)$
B_1	$(-2,6)$	(x,y)

Consider also strategy profile $s = (s_1, s_2)$

$$s_i(h) = \begin{cases} B_i & \text{if } h \in (B_1, B_2)^* \\ A_i & \text{otherwise} \end{cases}$$

Find all pairs $(x, y) \in \mathbb{R}^2$ for which the minimal discount required for s to be an SPE is equal to $\frac{3}{5}$. Formally: find all the pairs $(x, y) \in \mathbb{R}^2$ such that $\inf\{\delta \in (0, 1) \mid s \text{ is SPE in } G^{\delta}_{irep}\} = \frac{3}{5}$.

Justify your reasoning.