CVIKO 2

(1)

P: 5 a) $f(x,y) = \dfrac{x-y}{x^2+y^2-1}$

v $D_f = \mathbb{R}^2 - k([0,0],1)$ zjavne body nespojitosti nie sú.
Je možné spojito dodefinovať f v $k([0,0],1)$?

Pretože $\dfrac{const \neq 0}{0} = \pm\infty$, má zmysel uvažovať iba body, kde
$x-y=0$ a súčasne $x^2+y^2=1$
teda $\left(\pm\frac{1}{\sqrt2}, \pm\frac{1}{\sqrt2}\right)$

Počítajme

$$\lim_{(x,y)\to\left(\frac{1}{\sqrt2},\frac{1}{\sqrt2}\right)} \frac{x-y}{x^2+y^2-1} = \left|\frac{0}{0}\right|.$$ Ukážeme, že neexistuje.

zvoľme $x=\frac{1}{\sqrt2}$, potom

$$\lim_{y\to\frac{1}{\sqrt2}} \frac{\frac{1}{\sqrt2}-y}{y^2-\left(\frac{1}{\sqrt2}\right)^2} = \lim_{y\to\frac{1}{\sqrt2}} -\frac{1}{y+\frac{1}{\sqrt2}} = -\frac{1}{\sqrt2}$$

$y=\frac{1}{\sqrt2}$

$$\lim_{x\to\frac{1}{\sqrt2}} \frac{x-\frac{1}{\sqrt2}}{x^2-\left(\frac{1}{\sqrt2}\right)^2} = \lim_{x\to\frac{1}{\sqrt2}} \frac{1}{x+\frac{1}{\sqrt2}} = \frac{1}{\sqrt2}$$

t.j. limita neexistuje

Podobne

$x=-\frac{1}{\sqrt2}$

$$\lim_{y\to-\frac{1}{\sqrt2}} \frac{-\frac{1}{\sqrt2}-y}{y^2-\left(\frac{1}{\sqrt2}\right)^2} = -\lim_{y\to-\frac{1}{\sqrt2}} \frac{1}{y-\frac{1}{\sqrt2}} = \frac{1}{\sqrt2}$$ ~~neexistuje~~

$\neq$ NEEXISTUJE

$y=-\frac{1}{\sqrt2}$

$$\lim_{x\to-\frac{1}{\sqrt2}} \frac{x+\frac{1}{\sqrt2}}{x^2-\left(\frac{1}{\sqrt2}\right)^2} = \lim_{x\to-\frac{1}{\sqrt2}} \frac{1}{x-\frac{1}{\sqrt2}} = -\frac{1}{\sqrt2}$$

Pr 1 $c(t) = (\ln(t), \operatorname{arctan}(t), e^{\sin(\pi t)})$ $t_0 = 1$

$y(t) = c(t_0) + (t - t_0) \cdot \boxed{c'(t_0)}$ → smerový vektor

$c(1) = (0, \frac{\pi}{4}, 1)$

$c'(t) = (\frac{1}{t}, \frac{1}{1+t^2}, e^{\sin(\pi t)} \cdot \cos(\pi t) \cdot \pi)$

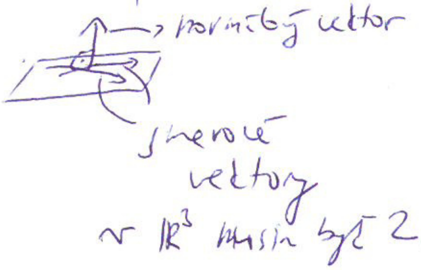
$c'(1) = (1, \frac{1}{2}, -\pi)$

$y(t) = (0, \frac{\pi}{4}, 1) + (t-1)(1, \frac{1}{2}, -\pi)$

Pr 2 $c(t) = (t^2 - 1, -2t^2 + 5t, t - 5)$ $\rho: \underline{\underline{3}}x + \underline{\underline{}}y - \underline{\underline{}}z + \boxed{7} = 0$

rovina je daná normálovým vektorom až na posunutie.

$\vec{n} = (3, 1, -1)$



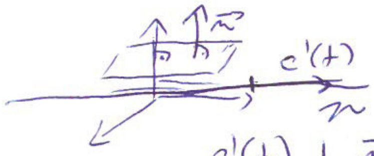
smerový vektor tečny v t je $c'(t)$

t.j. $c'(t) = (2t, -4t + 5, 1)$

tečna má byť rovnobežná s ρ t.j.

t.j. $\vec{n} \perp c'(t)$

ich skalárny súčin je 0



$\langle (2t, -4t+5, 1), (3, 1, -1) \rangle = 2t \cdot 3 + (-4t+5) \cdot 1 + 1 \cdot (-1)$

$= 6t - 4t + 5 - 1 = 2t + 4 = 0$

$\Rightarrow \boxed{t_0 = -2}$

Hľadaný bod → $\boxed{c(t_0) = (+3, -18, -7)}$

Pr 3 $f(x, y) = x^3 + 4xy$

$[x_0, y_0] = [2, -1]$ $d_u f = f'_u(x_0, y_0) = \lim_{t \to 0} \frac{f(x_0 + t \cdot u_1, y_0 + t u_2) - f(x_0, y_0)}{t}$

$u = (1, 3)$

$\lim_{t \to 0} \frac{(2+t)^3 + 4(2+t)(-1+3t) - \cancel{8} + \cancel{8}}{t} =$

$= \lim_{t \to 0} \frac{2^3 + t \cdot 3 \cdot 2^2 - \cancel{42} - 4t + 24t}{t} = 32$

Zaujímajú nás rovnice t menšie ako 2, lebo pre $n \geq 2$ máme $\lim_{t \to 0} a \frac{t^n}{t} = \lim_{t \to 0} a t^{n-1} = 0$.

Ak f má spojité parciálne derivácie v okolí $[x_0, y_0]$, potom má v $[x_0, y_0]$ diferenciál a platí

$d_u f(x) = \sum_{i=1} \frac{\partial f}{\partial x_i}(x) u_i$ LIN. ZOBRAZENIE

$f'_x = 3x^2 + 4y \xrightarrow{(2,-1)} 12-4=8$

$f'_y = 4x \xrightarrow{(2,-1)} 8$

$d_u f(2,-1) = 8 u_1 + 8 \cdot u_2$

$= 8 + 3 \cdot 8 = 4 \cdot 8 = \underline{\underline{32}}$

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④ $\operatorname{arctg}(x^2+y^2)$ $[x_0,y_0] = [1,-1]$ $u = (1,2)$

spojité na okolí $[1,-1]$ (rozšíř.) !

$$f'_x = \frac{1}{1+(x^2+y^2)^2} \cdot 2x$$

$$f'_y = \frac{1}{1+(x^2+y^2)^2} \cdot 2y$$

$d_u f(x_0,y_0) = f'_x(x_0,y_0) \cdot u_1 + f'_y(x_0,y_0) u_2$

$= \frac{2}{5} u_1 - \frac{2}{5} u_2$

diferenciál na okolí $[1,-1]$ je

$u \longmapsto d_u f(1,-1) = \frac{2}{5} u_1 - \frac{2}{5} u_2$

$$d_{(1,2)} f(-1,1) = \frac{2}{5} - \frac{4}{5} = -\frac{2}{5}$$

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⑤ $f(x,y) = \sqrt{x^2+y^2}$ $[x_0,y_0] = [3,4]$ u_1 u_2

$$z(x,y) = f(x_0,y_0) + \underbrace{f'_x(x_0,y_0)(x-x_0) + f'_y(x_0,y_0)}_{d_{(x-x_0,\,y-y_0)} f(x_0,y_0)}(y-y_0)$$

$f'_x(x,y) = \frac{1}{2}\frac{1}{\sqrt{x^2+y^2}} \cdot 2x \xrightarrow{(3,4)} \frac{1}{5} \cdot 3$

$f'(x,y) = \frac{1}{2}\frac{1}{\sqrt{x^2+y^2}} 2y \xrightarrow{(3,4)} \frac{4}{5}$

$f(x,y) = 5 + \frac{3}{5}(x-3) + \frac{4}{5}(y-4)$

$f(2{,}98; 4{,}05) \approx z(2{,}98; 4{,}05) = 5 - \frac{3}{5} \cdot 0{,}020 + \frac{4}{5} \cdot 0{,}05$

$= 5 - 3 \cdot 0{,}004 + 0{,}04$

$= 5 - 0{,}012 + 0{,}04$

$= 5{,}028$

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⑥ $f(x,y) = \operatorname{arctg}\frac{x}{y}$ $[x_0,y_0] = [1,1]$

$$f'_x(x,y) = \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \frac{1}{y}$$

$$f'_y(x,y) = \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \frac{-x}{y^2}$$

$z(x,y) = \frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{2}(y-1)$ $0{,}035$

$= \frac{\pi}{4} + 0{,}01 + 0{,}025$

$f(1{,}02; 0{,}95) \approx z(1{,}02; 0{,}95) = \frac{\pi}{4} + \frac{1}{2} 0{,}02 + \frac{1}{2} 0{,}05$

$f(x,y) = x^2 + xy + 2y^2 \quad v \quad [1,1, \overset{?}{=} 4]$ (4)

$f'_x(x,y) = 2x + y \overset{(1,1)}{\rightsquigarrow} 3 \qquad z(x,y) = 4 + 3(x-1) + 5(y-1)$

$f'_y(x,y) = x + 4y \overset{(1,1)}{\rightsquigarrow} 5$

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(8) $F(x,y,z) = (\underbrace{x^2 + y^2 + z^2}_{f(x,y,z)}, \overset{g(x,y,z)}{xyz})$ v bodě $[1,2,3]$

spojité f_x' ... spojité f.: $\underset{(1,2,3)}{DF}: v \longmapsto D_v F(1,2,3) = DF(1,2,3) \cdot v$

$$DF(x_0,y_0,z_0) = \begin{pmatrix} f'_x(x_0,y_0,z_0), & f'_y(x_0,y_0,z_0) & f'_z(x_0,y_0,z_0) \\ g'_x(x_0,y_0,z_0) & g'_y(x_0,y_0,z_0) & g'_z(x_0,y_0,z_0) \end{pmatrix}$$

$f'_x = 2x, \quad f'_y = 2y, \quad f'_z = 2z \qquad DF(1,2,3) = \begin{pmatrix} 2 & 4 & 6 \\ 6 & 3 & 2 \end{pmatrix}$

$g'_x = yz, \quad g'_y = xz, \quad g'_z = xy$
