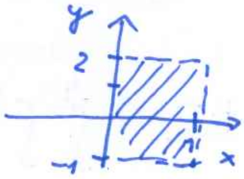
Pr 1

DVOJNÝ INTEGRÁL — DVOJNÁSOBNÝ INTEGRÁL

$$\iint_{[0,1]\times[-1,2]} x^2+2xy \, dx\,dy = \int_0^1\int_{-1}^{2} x^2+2xy \, dy\,dx = \int_0^1 \left[x^2y + xy^2\right]_{-1}^{2} dx$$

$$= \int_0^1 3x^2+3x \, dx = \left[x^3 + \frac{3}{2}x^2\right]_0^1 = 1+\frac{3}{2} = \frac{5}{2}$$

oblasť:



Pr 2

$$\iint_{[0,1]\times[0,3]} 3(x-1)^2+(y-2)^2+2 \, dx\,dy =$$

$$= \int_0^1\int_0^3 3(x-1)^2+(y-2)^2+2 \, dy\,dx =$$

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$$= \int_0^1 \left[3(x-1)^2y + \frac{(y-2)^3}{3} + 2y\right]_0^3 dx =$$

Pripomenutie: (Jednoduchá substitúcia)

$$\int (x-c)^2 \, dx = \left| \begin{matrix} y = x-c \\ dy = dx \end{matrix} \right|$$

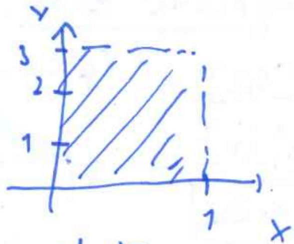
$$\int y^2 dy = \frac{y^3}{3}$$

$$= \frac{1}{3}\left((x-c)^3\right)$$

$$= \int_0^1 9(x-1)^2 + \frac{1}{3} + \frac{8}{3} + 6 \, dx = 9\int_0^1 (x-1)^2 + 1 \, dx$$

$$= 9\left[\frac{(x-1)^3}{3} + x\right]_0^1 = 9\left(1+\frac{1}{3}\right) = 12$$

oblasť:

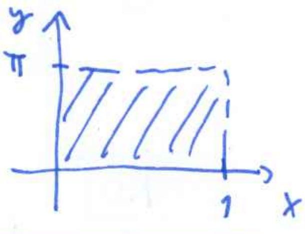


Pr 4

$$\int_0^1\int_0^\pi x\sin y \, dy\,dx = \int_0^1 -\left[x\cos(y)\right]_0^\pi dx$$

$$= -\int_0^1 -x-x \, dx = \int_0^1 2x\,dx = \left[x^2\right]_0^1 = 1$$

oblasť:

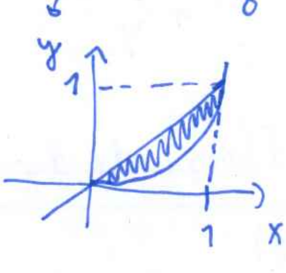


Pr 3

$$\int_0^1\int_{x^2}^{x} (2-xy)\,dy\,dx = \int_0^1 \left[2y - x\frac{y^2}{2}\right]_{x^2}^{x} dx = \int_0^1 2x - \frac{x^3}{2} - 2x^2 + \frac{x^5}{2} \, dx$$

$$= \left[x^2 - \frac{x^4}{8} - \frac{2}{3}x^3 + \frac{x^6}{12}\right]_0^1 = 1 - \frac{1}{8} - \frac{2}{3} + \frac{1}{12} = \frac{24-3-16+2}{24} = \frac{7}{24}$$

oblasť:



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Pr 5 $\displaystyle\int_3^4\int_x^{2x} \frac{1}{x^2-3x+2}\,dy\,dx = \int_3^4 \frac{1}{x^2-3x+2}\cdot[y]_x^{2x}\,dx = \int_3^4 \frac{x}{x^2-3x+2}\,dx =$

ROZKLAD NA PARCIÁLNE ZLOMKY

$$\frac{x}{x^2-3x+2} = \frac{A}{(x-2)} + \frac{B}{(x-1)} = \frac{A(x-1)+B(x-2)}{(x-2)(x-1)}$$

$x=1$: $1=-B$

$x=2$: $2=A$ $\Rightarrow$ $\boxed{\frac{2}{x-2} - \frac{1}{x-1}} = \frac{x}{x^2-3x+2}$

$$= \int_3^4 \frac{2}{x-2} - \frac{1}{x-1}\,dx = \Big[2\ln|x-2| - \ln|x-1|\Big]_3^4 = 2\ln 2 - \ln 3 - 2\underbrace{\ln 1}_{=0} + \ln 2$$

$$= \underline{3\ln 2 - \ln 3}$$

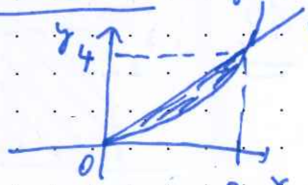
Pr 6 Zameňte poradie integrácie $\displaystyle\int_0^2\int_{x^2}^{2x} f(x,y)\,dy\,dx$

integračná oblasť:

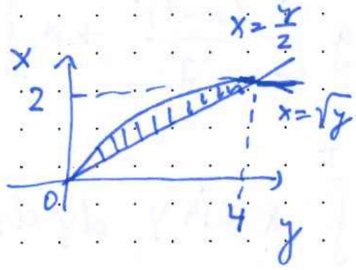
$x\in[0,2]$

$y\in[x^2,2x]$

$\boxed{x^2\le y\le 2x}$



„prekreslíme opačne" $\rightsquigarrow$



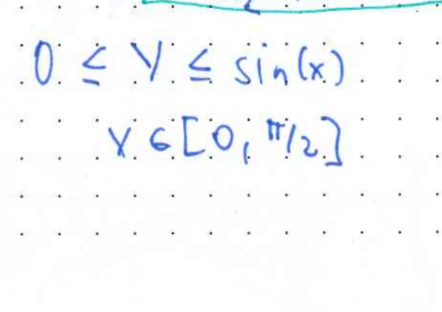
$y=x^2,\ x\ge 0$ $\Rightarrow$ $x=\sqrt{y}$

$y=2x$ $\Rightarrow$ $x=\frac{y}{2}$

$$\Rightarrow \int_0^4\int_{\frac{y}{2}}^{\sqrt{y}} f(x,y)\,dx\,dy$$

Pr 7 CVIKO

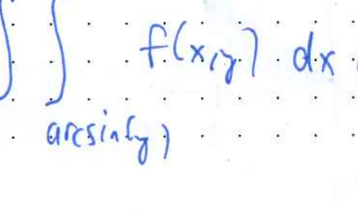
oblasť: $\displaystyle\int_0^{\pi/2}\int_0^{\sin x} f(x,y)\,dy\,dx$



$\rightsquigarrow$ dívame sa z iného uhla

$0\le y\le \sin(x)$

$x\in[0,\pi/2]$



$y=\sin x$ $\Rightarrow$ $x=\arcsin y$

$\arcsin(y)\le x\le \frac{\pi}{2}$

$y\in[0,1]$

$$\int_0^1\int_{\arcsin(y)}^{\pi/2} f(x,y)\,dx\,dy$$

Pr 8

$$\int_0^{\sqrt{\frac{\pi}{2}}} \int_y^{\sqrt{\frac{\pi}{2}}} y^2 \sin(x^2)\, dx\, dy$$

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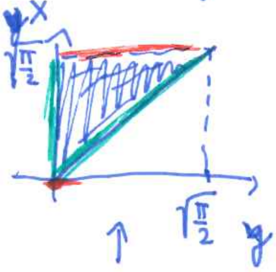
$\int \sin x^2\, dx$ nevieme spočítať

$\Rightarrow$ zámena integrálu

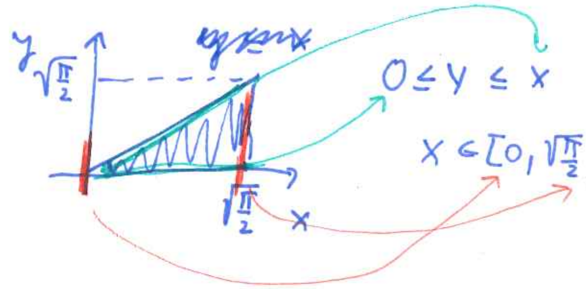


pohľad odtiaľto

po zámene poradia

$y \le x \le \sqrt{\frac{\pi}{2}}$

$y \in [0, \sqrt{\frac{\pi}{2}}]$ $\rightsquigarrow$



$0 \le y \le x$

$x \in [0, \sqrt{\frac{\pi}{2}}]$

$$\Rightarrow \int_0^{\sqrt{\frac{\pi}{2}}} \int_0^{x} y^2 \sin(x^2)\, dy\, dx = \int_0^{\sqrt{\frac{\pi}{2}}} \left[\frac{y^3}{3}\right]_0^x \sin(x^2)\, dx = \frac{1}{3}\int_0^{\sqrt{\frac{\pi}{2}}} x^3 \sin(x^2)\, dx$$

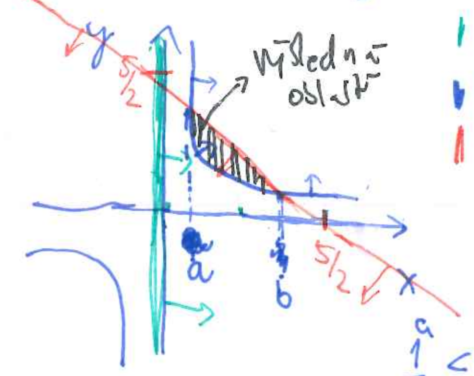
$$\left|\begin{array}{c} t = x^2 \\ dt = 2x\,dx \\ x > 0 \\ |2x| = 2x \\ 0 \longmapsto 0 \\ \sqrt{\frac{\pi}{2}} \longmapsto \frac{\pi}{2} \end{array}\right| = \frac{1}{3}\int_0^{\pi/2} t \cancel{x} \sin(t) \frac{1}{2\cancel{x}}\, dt = \frac{1}{6}\int_0^{\pi/2} \overset{u\cdot v'}{t \sin(t)}\, dt$$

$$\overset{P.P.}{=} \frac{1}{6}\left(\overset{u\cdot v}{\left[-t \cdot \cos(t)\right]_0^{\pi/2}} - \int_0^{\pi/2} \overset{u'\cdot v}{-\cos(t)}\, dt \right|$$

$$= \frac{1}{6}\left(0 + \left[\sin(t)\right]_0^{\pi/2}\right) = \underline{\underline{\frac{1}{6}}}$$

Pr 9

$$I = \iint 8y\, dx\, dy \qquad M = \left\{[x,y] \in \mathbb{R}^2 \mid x \ge 0,\ x\cdot y \ge 1,\ x + y \le 5/2\right\}$$



$x = 0$

$x \cdot y = 1 \Rightarrow y = \frac{1}{x}$

$x + y = 5/2$

$y \overset{\le}{=} 5/2 - x$

$x + y = 5/2$, $x \cdot y = 1 \Rightarrow y = \frac{1}{x}$

$x + \frac{1}{x} = 5/2$

$x^2 + 1 = 5/2\, x$

$x^2 - 5/2\, x + 1 = 0 \qquad D = \frac{25}{4} - 4 = \frac{9}{4}$

$x_1, x_2 = \frac{5/2 \pm 3/2}{2} \begin{cases} 2 = b \\ 1/2 = a \end{cases}$

$\overset{a}{\frac{1}{2}} \le x \le \overset{b}{2}$

$\frac{1}{x} \le y \le 5/2 - x$

$$\int_{1/2}^{2} \int_{1/x}^{5/2 - x} 8y\, dy\, dx = 8\int_{1/2}^{2} \left[\frac{y^2}{2}\right]_{1/x}^{5/2 - x} dx = 4\int_{1/2}^{2} (5/2 - x)^2 - \frac{1}{\overset{x^{-2}}{x^2}}\, dx = 4 \cdot \left[-\frac{(5/2 - x)^3}{3} - \frac{x^{-1}}{-2+1}\right]_{1/2}^{2}$$

$$\boxed{\frac{9}{2} = \frac{27}{6} = \frac{27}{24}\cdot 4}$$

$$= 4\left(-\frac{1}{24} + \frac{1}{2} + \frac{2^3}{3} - 2\right) = \frac{-1 + 12 + 8\cdot 2^3 - 48}{24} \cdot 4$$