Bernoulliho D.R $\qquad y' = y \cdot p(x) + y^r \cdot q(x)\ , \ \underline{\underline{r > 1}}$

Vždy riešenie $y = 0$, substitúcia $z = y^{1-r}$

(5) $y' = \frac{y}{x} + y^2 \sin(x)$ $\qquad$ pre nás $z = y^{-1}$ $\quad z' = -\frac{1}{y^2} \cdot y' \Rightarrow y' = -y^2 z'$

$$-y^2 z' = \frac{y}{x} + y^2 \sin(x) \quad \Big| : y^2 \qquad IF = e^{\int \frac{1}{x} dx} = e^{\ln|x|} = |x|$$

$$-z' = \frac{1}{y} \frac{1}{x} + \sin(x)$$

$$-z' - \frac{z}{x} = \sin(x)$$

$$-\left(z' + \frac{z}{x}\right) = \sin(x) \quad / \, x$$

$$-(z'x + z \cdot 1) = x \cdot \sin(x)$$

$$-(z \cdot x)' = x \cdot \sin(x) \quad / \int$$

$$-z \cdot x = \int x \cdot \sin(x)\, dx$$

$$= x \cdot (-\cos(x)) + \int \cos(x)\, dx = x(-\cos(x)) + \sin(x) + c$$

môžeme zobrať ľubovoľný násobok a ľubovoľnú primitívnu funkciu, preto vezmeme x.

$$\boxed{z = \cos(x) - \frac{\sin(x)}{x} + \frac{c}{x}} \qquad \frac{1}{y} = \cos(x) - \frac{\sin(x)}{x} + \frac{c}{x}$$

$y(1) = 4$ $\qquad \frac{1}{4} = \cos(1) - \sin(1) + c \Rightarrow \underline{\underline{c = \frac{1}{4} - \cos(1) + \sin(1)}}$

(1) $y'' = 2y' + y + 1$ Najskôr vyriešime homogénnu DR t.j. $y'' = 2y' + y + \underline{0}$

Characteristická rovnica: $y^{(n)}$ nahradíme λ^n

$\lambda^2 = 2\lambda + 1$ $\quad \lambda^2 - 2\lambda - 1 = 0 \quad D = 4 + 4 = 8 \Rightarrow \frac{2 \pm 2\sqrt{2}}{2} < \underline{1 \pm \sqrt{2}}$

riešenie homogénnej DR.: $y_H = c_1 \cdot \exp((1+\sqrt{2})x) + c_2 \cdot \exp((1-\sqrt{2})x)$

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riešenie nehomogénnej DR: $y'' - 2y' - y = 1 e^{0 \cdot x}$

0 nie je koreňom charakteristickej rovnice polynómu $\lambda^2 = 2\lambda + 1$,

preto píšeme $y_p = A \cdot e^{0x}$ $\quad y_p' = 0 \quad y_p'' = 0$

$$0 - 2 \cdot 0 - A \cdot e^{0x} = 1 \cdot e^{0 \cdot x} \Rightarrow \underline{A = -1}$$

Celkové riešenie je $y_H + y_P = y$ („princíp superpozície")

$$y = c_1 \exp((1+\sqrt{2})x) + c_2 \cdot \exp((1-\sqrt{2})x) - 1$$

Riešenie spĺňajúce $y(0) = 0$, $y'(0) = 1$

$$y' = y_H' = (1+\sqrt{2}) c_1 \exp((1+\sqrt{2})x) + (1-\sqrt{2}) c_2 \exp((1-\sqrt{2})x)$$

$$y'(0) = (1+\sqrt{2}) c_1 + (1-\sqrt{2}) c_2 = 1$$

$$y(0) = c_1 + c_2 - 1 = 0 \Rightarrow \underline{c_1 = 1 - c_2}$$

už odtiaľto vieme uhádnuť, že riešenie je $(c_1, c_2) = (1/2, 1/2)$

$$(1+\sqrt{2})(1 - c_2) + (1-\sqrt{2}) c_2 = \cancel{1}$$

$$(-2\sqrt{2}) \cdot c_2 = -\sqrt{2} \Rightarrow c_2 = \frac{1}{2},\ c_1 = 1/2$$

Celkovo $y = \frac{1}{2} \exp((1+\sqrt{2})x) + 1/2 \exp((1-\sqrt{2})x) - 1$

Skúška správnosti: $y'' = y_H'' = (1+\sqrt{2})^2 c_1 \exp((1+\sqrt{2})x) + (1-\sqrt{2})^2 c_2 \exp((1-\sqrt{2})x)$

VŠETKO SPRÁVNE:

Dosadíme do zadania: $3(c_1 \exp((1+\sqrt{2})x) + c_2 \exp((1-\sqrt{2})x)) + 2\sqrt{2}(c_1 \exp((1+\sqrt{2})x) - c_2 \exp(($

$= 2((1+\sqrt{2}) c_1 \exp((1+\sqrt{2})x) + (1-\sqrt{2}) c_2 \exp((1-\sqrt{2})x)) + c_1 \exp((1+\sqrt{2})x) + c_2 \exp((1-\sqrt{2})x)) - 1 + 1$

Zadanie: $y'' + 3y' + 2y = (x+1)(e^{-3x} + e^{-x})$

(2) charakt. rovnica

$$y = y_H + y_{p_1} + y_{p_2}$$

$$K(\lambda) = \lambda^2 + 3\lambda + 2 = 0 \qquad \lambda = \begin{cases} -1 \\ -2 \end{cases}$$

$$(\lambda+2)(\lambda+1) = 0$$

$$y_H = c_1 e^{-x} + c_2 e^{-2x}$$

$$y_{p_1} = (Ax+B)\cdot e^{-3x}$$, lebo $K(-3) \neq 0$

dosadíme do zadania

$$y'_{p_1} = A e^{-3x} + (Ax+B)(-3)\cdot e^{-3x}$$

$$y''_{p_1} = -3Ae^{-3x} - 3Ae^{-3x} + (Ax+B)(-3)^2\cdot e^{-3x} = -6Ae^{-3x} + (Ax+B)\,9e^{-3x}$$

$$-6Ae^{-3x} + (Ax+B)\cdot 9e^{-3x} + 3\left(Ae^{-3x} + (Ax+B)(-3)e^{-3x}\right) + 2(Ax+B)e^{-3x} = (x+1)e^{-3x}$$

x: $\cancel{9A} - \cancel{9A} + 2A = 1 \Rightarrow A = 1/2$

$1 = x^0$: $-6A + \cancel{9B} + 3A - \cancel{9B} + 2B = 1 \qquad -3A + 2B = 1$

$$-3/2 + 2B = 1 \Rightarrow B = \frac{5}{4}$$

$$y_{p_1} = (x + 5/4)\cdot e^{-3x}$$

$y_{p_2} = x^1\cdot(Cx+D)e^{-x}$, pretože -1 je 1-násobný koreň charakteristickej rovnice.

$$y'_{p_2} = (Cx+D)e^{-x} + xCe^{-x} - x(Cx+D)e^{-x}$$

$$y''_{p_2} = Ce^{-x} - (Cx+D)e^{-x} + Ce^{-x} - xCe^{-x} - (Cx+D)e^{-x} - xCe^{-x} + x(Cx+D)e^{-x}$$

$$y''_{p_2} + 3y'_{p_2} + 2y_{p_2} = (x+1)e^{-x}$$

x^2: $Cx^2 - 3Cx^2 + 2Cx^2 = 0$ platí pre každé C (musí vyjsť vždy)

x: $-Cx - Cx - Cx - Cx + Dx + 3(Cx + Cx - Dx) + 2Dx = x$

$$2Cx = x \Rightarrow C = 1/2$$

x^0: $C - \cancel{D} + C - \cancel{D} + 3(\cancel{D}) + 2(0) = 1$

$$2C + D = 1 \Rightarrow D = 0$$

$$y_{p_2} = x^2 e^{-x}$$

$$\boxed{y = y_H + y_{p_1} + y_{p_2}}$$

(4)

$$\lambda^2+2\lambda+2=0 \qquad D=4-4\cdot 2=-4<0$$

$$\lambda_{1,2}=\frac{-2\pm i2}{2} < \begin{matrix}-1+i\\-1-i\end{matrix}$$

$$y_H = e^{-x}(c_1\cos x + c_2\sin x)$$

1-násobný

$$b(x)=3e^{-x}\cos x = 3e^{-x}\cdot\cos(1x) \qquad -1+i \text{ je koreň } \lambda^2+2\lambda+2=0$$

stupeň je maximálny z $b(x)$

$$y_p = x\cdot e^{-x}\cdot(C\sin(x)+B\cos(x))$$

$$y_p' = e^{-x}(C\sin x+\boxed{B\cos x}) - x e^{-x}(C\sin(x)+B\cos(x))$$
$$+x\cdot e^{-x}(C\cos(x) - B\sin(x)) \to e^{-x}(\boxed{C\cos x} - B\sin x)$$

$$y_p'' = -e^{-x}(C\sin x+\boxed{B\cos x}) - e^{-x}(C\sin(x)+\boxed{B\cos(x)})$$
$$+x e^{-x}(C\sin(x)+B\cos(x)) - x e^{-x}(C\cos(x)-B\sin(x))$$
$$+e^{-x}(\boxed{C\cos(x)}-B\sin(x)) - x e^{-x}(C\cos(x)-B\sin(x))$$
$$+x\cdot e^{-x}(-C\sin(x)-B\cos(x))$$

$$y_p''+2y_p'+2y_p = e^{-x}(3\cos(x))$$

$e^{-x}x^1\cos(x)$:

$$B-C-C-\cancel{B}+2(C-\cancel{B})+2\cancel{B}=0 \checkmark$$ (musí vždy platiť, pre každé C, D !)

$e^{-x}x^1\sin(x)$: $\cancel{C}+\cancel{B}+\cancel{B}-\cancel{C}+2(-\cancel{B}-\cancel{C})+2\cancel{C}=0$

$e^{-x}\cos(x)$: $C-\cancel{B}-\cancel{B}+C+2\cancel{B}+2\cdot 0=3$

$$2C=3 \Rightarrow C=\frac{3}{2}$$

$e^{-x}\sin(x)$: $-B-\cancel{C}-\cancel{C}+\cancel{B}+2\cancel{C}+2\cdot 0=0$

$$-2B=0 \Rightarrow B=0$$

$$y_p=\frac{3}{2}x e^{-x}(\sin(x))$$

y_p je tvaru $x^s\cdot e^{\alpha x}\big(\underset{\max\{m,n\}}{P(x)}\cos(\beta x)+\underset{\max\{m,n\}}{Q(x)}\sin(\beta x)\big)$,

ak $b(x)=e^{\alpha(x)}\cdot(\tilde{P}_m(x)\cos(\beta x)+\tilde{Q}_n\sin(\beta x))$

s = násobnosť koreňa $\alpha+i\beta$ charakteristickej rovnice