

# Logical analysis of natural language

Marek Medved'

Natural Language Processing Centre  
Faculty of Informatics, Masaryk University  
Botanická 68a, 602 00 Brno  
`xmedved1@fi.muni.cz`

PA153 Natural Language Processing

# Outline

- 1 Motivation
- 2 Predicate logic
- 3 Transparent intensional logic
- 4 Propositional logic

# Logical analysis of natural language

- Sentence  $\rightarrow$  logical formula
  - formal reasoning
  - interlingua for machine translation
  - precise expressions
- Which formalism?
  - (first order) predicate logic
  - propositional logic
  - modal logics
  - intensional logics (IL, Richard Montague)
  - transparent intensional logic (TIL)

# Natural language $\rightarrow$ predicate logic

## ■ What are formulas for

- „Some prime numbers are even”
- „Some odd numbers are even”
- „Some smart people are lazy”
- „No bachelor is married”
- „No bachelor is rich”
- „Miloš Zeman is the president of CR.”
- „Karel counts  $5 + 7$ ”

## ■ What is wrong?

# Natural language $\rightarrow$ predicate logic

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## ■ What is wrong?

- different types of truth
- granularity of the description is insufficient

# Natural language $\rightarrow$ predicate logic

Karel counts  $5 + 7$

$$5 + 7 = 12$$

---

Karel counts 12

Miloš Zeman is the president of CR.

Karel Schwarzenberg wanted to become the president of CR.

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Karel Schwarzenberg wanted to become Miloš Zeman.

It is not true that the king of France is bald-headed

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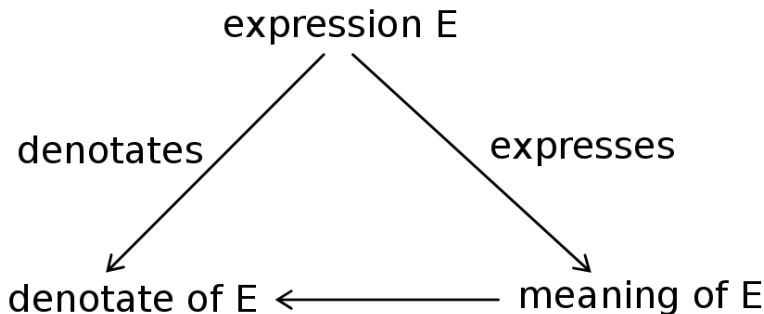
The king of France has some hair on his head.

# Natural language $\rightarrow$ predicate logic

- Predicate logic can be used for natural language analysis
  - some applications are doing it
  - but not in all cases
  - we need to be aware of the limits
  - work-arounds are possible but may be complicated
- Advantages of predicate logic
  - it is simple
  - it is well explored
  - inference machine exists

# What is the meaning of an expression?

Frege's model of semantics





# Conception of possible worlds

## ■ A possible world

- a set of non-contradictory formulas about the universe
- the current world is one of the possible worlds

## ■ Empirical truth

- the truth of a formula depends on the particular world
- meaning is always world-independent

## ■ Intensional logics

- intensions (world-independent)
- extensions (denotes, objects in a particular world)

# Transparent intensional logic

- Pavel Tichý, Pavel Materna

- Procedural logic

- possible worlds + possible times
- meaning is a construction, i.e. abstract procedure (algorithm) which takes the current world and time and outputs the denotation (the particular object)
- coded as lambda function

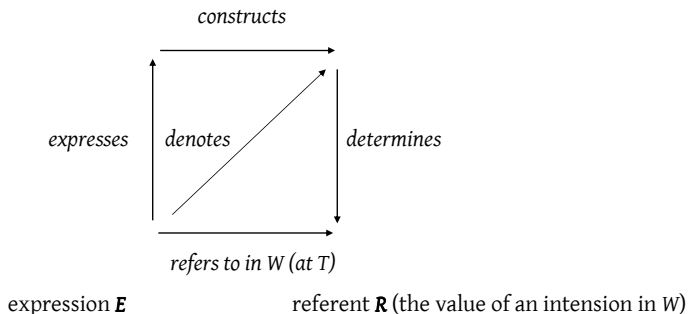
- Typed logic

- $o$  – true, false
- $\iota$  – set of individuals
- $\tau$  – set of real numbers (or time moments)
- $\omega$  – set of possible worlds
- $((o\tau)\omega)$  – proposition
- $((((o\iota)\tau)\omega)$  – property  $((o\iota)_{\tau}\omega)$

## Tichý model of semantics

meaning **M** (construction)

denotatum **D** (intension/extension)



# Transparent intensional logic – examples

Miloš Zeman je prezidentem CR.

$\lambda w \lambda t [ = \text{Miloš\_Zeman} \text{ President\_CR}_{wt} ]$

$\text{type } o_{\tau\omega} \quad \text{Miloš\_Zeman}/\iota \quad \text{President\_CR}/\iota_{\tau\omega} \quad = / (o\iota\iota)$

Schwarzenberg wanted to become CR.

$\lambda w \lambda t [ \text{want\_to\_become}_{wt} \text{ Schwarzenberg} \text{ President\_CR} ]$

$o_{\tau\omega} \quad \text{Schwarzenberg}/\iota \quad \text{President\_CR}/\iota_{\tau\omega} \quad \text{want\_to\_become}/(o\iota\iota_{\tau\omega})$

# Transparent intensional logic – examples

$5 + 7$

$[+ 5 7]$

type  $\tau$        $5, 7/\tau$        $+ /(\tau\tau\tau)$

Karel counts  $5 + 7$ .

$\lambda w \lambda t [\text{count}_{wt} \text{ Karel } ^0 [+ 5 7]]$

type  $o_{\tau\omega}$        $^0 [+ 5 7]/*_1$        $\text{count}/(o\iota*_1)_{wt}$        $\text{Karel}/\iota$

# Normal translation algorithm

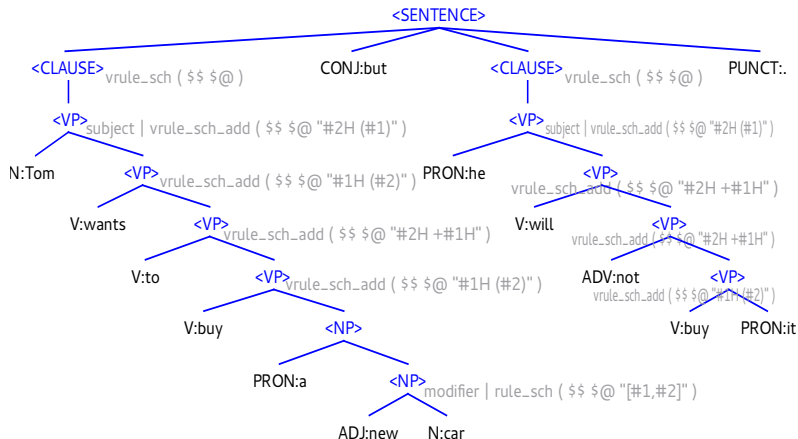
## ■ Automatic conversion of sentences into TIL constructions

- doc. Aleš Horák
- morphological analysis
- syntactic analysis
- conversion from tree to TIL formula
- type check
- output of all the layers is ambiguous
- implementation within the Synt parser, currently experiments with other parsers (SET)

## ■ Further requirements

- lexicon of types (“snow” vs. “give” – exploitation of valency lexicons)
- rules for type control

# Example



# Example

$$\begin{aligned}
 & \lambda w_1 \lambda t_2 \left( (\exists x_3) (\exists i_4) (\exists i_5) \left( [\text{Does}_{w_1 t_2}, i_5, [\text{Imp}_{w_1}, x_3]] \wedge [[\text{new, car}]_{w_1 t_2}, i_4] \right. \right. \\
 & \wedge x_3 = [\text{to\_want}, i_4]_{w_1} \wedge [\text{Tom}_{w_1 t_2}, i_5] \Big) \wedge [\text{Not}, [\text{True}_{w_1 t_2}, \\
 & \lambda w_6 \lambda t_7 (\exists x_8) (\exists i_9) \left( [\text{Does}_{w_6 t_7}, \text{He}, [\text{Perf}_{w_6}, x_8]] \wedge [\text{it}_{w_6 t_7}, i_9] \right. \\
 & \left. \left. \left. \wedge x_8 = [\text{to\_buy}, i_9]_{w_6} \right) \right] \right) \dots \pi
 \end{aligned}$$



# Transparent intensional logic – pros and cons

## ■ Advantages

- correct and very precise analysis
- makes general correct reasoning possible

## ■ Disadvantages

- very abstract and complex
- not really wide-spread
- experts often do not agree on correct analysis

# Transparent Intensional logic on FI

- doc. Aleš Horák
- prof. Marie Duží
  - subjects Introduction to Transparent Intensional Logic
- Small corpus of correct constructions for Czech
  - <https://corpora.fi.muni.cz/til>
- Semantic network of constructions
  - as a knowledge base for automatic reasoning

# Propositional logic

- used in logic programming in for of Horn clauses

$$T \leftarrow M$$

- can be used in Inductive Concept learning

# Inductive concept learning

- aims to learn an **intensional description** by induction from positive, negative examples and background knowledge
- induced description is also called hypothesis

# Inductive concept learning

$$\mathcal{E}^+ = \left\{ \begin{array}{l} \text{transport\_by\_land}(\text{bike}). \\ \text{transport\_by\_land}(\text{motorbike}). \\ \text{transport\_by\_land}(\text{car}). \\ \text{transport\_by\_land}(\text{jeep}). \\ \text{transport\_by\_land}(\text{truck}). \\ \text{transport\_by\_land}(\text{bus}). \\ \text{transport\_by\_land}(\text{hovercraft}). \end{array} \right.$$

$$\mathcal{E}^- = \left\{ \begin{array}{l} \text{transport\_by\_land}(\text{airplane}). \\ \text{transport\_by\_land}(\text{seaplane}). \\ \text{transport\_by\_land}(\text{airship}). \\ \text{transport\_by\_land}(\text{helicopter}). \end{array} \right.$$

$$\mathcal{B} = \left\{ \begin{array}{l} \text{has\_propeller}(\text{hovercraft}). \\ \text{has\_propeller}(\text{airplane}). \\ \text{has\_propeller}(\text{seaplane}). \\ \text{has\_propeller}(\text{helicopter}). \\ \text{has\_propeller}(\text{airship}). \\ \\ \text{has\_steering\_wheel}(\text{car}). \\ \text{has\_steering\_wheel}(\text{truck}). \\ \text{has\_steering\_wheel}(\text{bus}). \\ \text{has\_steering\_wheel}(\text{jeep}). \\ \\ \text{travels\_on\_wheels}(\text{motorbike}). \\ \text{travels\_on\_wheels}(\text{bike}). \\ \\ \text{vertical\_take\_off}(\text{helicopter}). \\ \text{vertical\_take\_off}(\text{airship}). \\ \\ \text{has\_wings}(\text{airplane}). \\ \text{has\_wings}(\text{seaplane}). \\ \\ \text{travels\_on\_wheels}(X) \leftarrow \text{has\_steering\_wheel}(X). \end{array} \right.$$

# Inductive concept learning

$$\mathcal{H}_B^{\mathcal{E}} = \begin{cases} \textit{transport\_by\_land}(X) \leftarrow \textit{travels\_on\_wheels}(X). \\ \textit{transport\_by\_land}(\textit{hovercraft}). \end{cases}$$

# Detecting outliers through concept learning

$$\mathcal{H}_B^{\mathcal{E}} = \begin{cases} \text{transport\_by\_land}(X) \leftarrow \text{travels\_on\_wheels}(X). \\ \text{transport\_by\_land}(\text{hovercraft}). \end{cases}$$

$$\mathcal{H}_B^{\mathcal{E} \setminus \mathcal{O}} = \{ \text{transport\_by\_land}(X) \leftarrow \text{travels\_on\_wheels}(X). \}$$

$$\mathcal{H}_B^{\bar{\mathcal{E}}} = \begin{cases} \text{not\_transport\_by\_land}(X) \leftarrow \text{has\_wings}(X). \\ \text{not\_transport\_by\_land}(X) \leftarrow \text{vertical\_take\_off}(X). \end{cases}$$

$$\mathcal{H}_B^{\bar{\mathcal{E}} \setminus \bar{\mathcal{O}}} = \{ \text{not\_transport\_by\_land}(X) \leftarrow \text{has\_propeller}(X). \}$$

- Exploiting domain knowledge to detect outliers (Fabrizio Angiulli · Fabio Fasseti)