Filters in Image Processing

Image Transforms (I)

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Outline

Definition:

Let $C \subset \mathbb{R}^{n \times n}$ be a square matrix. Vector $\mathbf{v} \in \mathbb{C}^n$ is an *eigenvector* of $C \Leftrightarrow \exists \lambda \in \mathbb{C} : C\mathbf{v} = \lambda \mathbf{v}$. λ is called an *eigenvalue*.

Properties of eigenvalues:

- $C\mathbf{v} = \lambda \mathbf{v}$ equals to $(C - \lambda E)\mathbf{v} = 0$
  The solution of equation $|C - \lambda E| = 0$ is identical to the search for roots in the polynomial of degree n.
- if C is symmetric then all the eigenvalues are real

Properties of eigenvectors:

- the two eigenvectors corresponding to two different eigenvalues are orthogonal
- if C is symmetric then all the eigenvectors are real and orthogonal

Properties of symmetric matrices:

- if $C \subset \mathbb{R}^{n \times n} : C = C^T$ with its eigenvectors $e_1, e_2, \dots, e_n$ and eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_n$ then $\exists A \subset \mathbb{R}^{n \times n}$ such that $A^T CA = D$, where

$$D = \begin{bmatrix} \lambda_1 & 0 & 0 & \dots & 0 \\ 0 & \lambda_2 & 0 & \dots & 0 \\ 0 & 0 & \lambda_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \lambda_n \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_n \end{bmatrix}$$

Cross-correlation:

- given two 1D signals $f(i)$ and $g(i)$:

$$(f \star g)(i) = \sum_k f(i+k)g(k)$$

- it is a measure of similarity of two signals

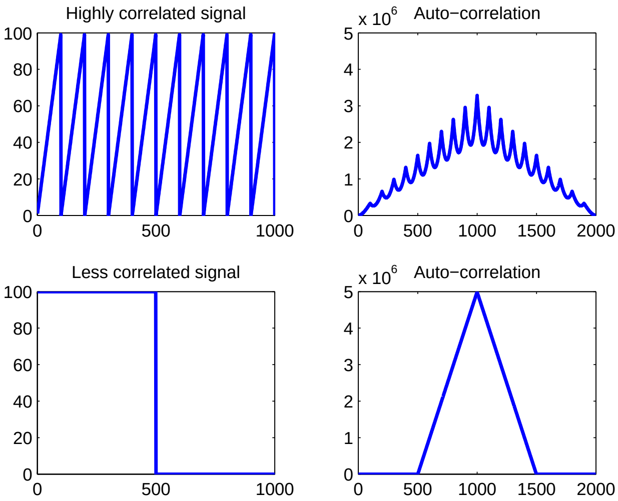
Auto-correlation:

- given 1D signal $f(i)$:

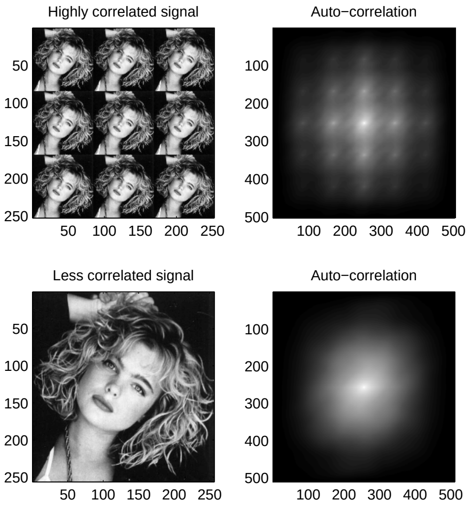
$$(f \star f)(i) = \sum_k f(i+k)f(k)$$

- it is a measure for finding repeating patterns

Auto-correlation example



Auto-correlation example



Highly correlated signals

- contain repeating patterns
- energy (nonzero values) is stretched over the whole space
- this is what we usually have

Decorrelated signals

- energy (nonzero values) is compacted in one location
- easy to compress
- this is what we wish to have

Covariance

- Covariance exhibits how much two signals f and g (let us assume $|f| = |g| = n$) vary from the mean with respect to each other:

$$cov(f,g) = \frac{\sum_{i=1}^{n}(f_i - \overline{f})(g_i - \overline{g})}{n-1}$$

- Covariance Matrix – a matrix of covariances between two signals f and g:

$$C = \begin{bmatrix} cov(f,f) & cov(f,g) \\ cov(g,f) & cov(g,g) \end{bmatrix}$$

Matrix C is always real and symmetric.

Notice: Two signals f and g are decorrelated iff $cov(f,g) = cov(g,f) = 0$, i.e. when the matrix C is diagonal.

Motivation

The most common linear transforms

- sinusoidal transforms
  - DFT (discrete Fourier transform)
  - DCT (discrete cosine transform)
  - DST (discrete sine transform)
- rectangular wave transforms
  - Walsh-Hadamard transform
  - Haar transform
- variable basis
  - Discrete wavelet transform
- eigenvector-based transforms
  - Karhunen-Loeve transform

Motivation
Energy compaction

Evaluate DFT over some short signal:

$$
\begin{aligned}
f &= [1\ 3\ 4\ 2] \\
&\Downarrow \quad /\text{DFT}/ \\
\mathcal{F} &= \frac{1}{\sqrt{4}}[10 \quad (-3-i) \quad 0 \quad (-3+i)]
\end{aligned}
$$

Measure the energy of the signals:

$$
\begin{aligned}
E(f) &= \sum f(i)^2 = 1^2 + 3^2 + 4^2 + 2^2 = 30 \\
E(\mathcal{F}) &= \sum \mathcal{F}(i)^2 = 25 + 10/4 + 10/4 = 30
\end{aligned}
$$

The first component in f accounts for 3.3% of energy while the first component in $\mathcal{F}$ accounts for 83.3% of energy.

Conclusion: Ability of energy compaction $\approx$ optimality of the transform

Motivation

Why do we need image transforms?

- manipulation with data in another domain might be simpler
  example of use: low-pass, high-pass filtering
- data decorrelation ≈ co-variance removal ≈ energy compaction
  example of use: image compression

Which transform properties are the most important?

- speed
- simple to implement

Notice: The requirements suggest to use *linear transforms*, i.e. the input signal f is transformed into the signal F by:

$$F = Af,$$

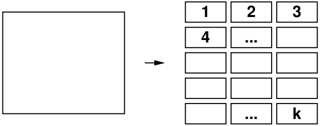
where A is a *transform matrix*.

Task to solve

Given an input discrete signal, design a transformation matrix such that the transformed signal has decorrelated samples (they are mutually independent).

Solution

Given a 2D image, let us break it up into k blocks of n pixels each.



Each block i is characterized with its vector $\mathbf{b}^{(i)}, i = 1, 2, \ldots, k$ where $\text{length}(\mathbf{b}^{(i)}) = n$.

PCA-based Transform

Design of transform matrix A

input (correlated)	input (mean centered)	expected output (decorrelated)
$\mathbf{b}^{(i)}$	$\mathbf{v}^{(i)} = \mathbf{b}^{(i)} - \overline{\mathbf{b}}$	$\mathbf{w}^{(i)} = A\mathbf{v}^{(i)}, i = 1, 2, \ldots, k$
	$V = \left[\mathbf{v}^{(1)}, \mathbf{v}^{(2)}, \ldots, \mathbf{v}^{(k)}\right]$	$W = AV = \left[\mathbf{w}^{(1)}, \mathbf{w}^{(2)}, \ldots, \mathbf{w}^{(k)}\right]$
	$C_V = V \cdot V^T$	$C_W = W \cdot W^T$
	C_V ... real, symmetric	C_W ... diagonal

$$C_W = W \cdot W^T = (AV) \cdot (AV)^T = A(V \cdot V^T)A^T = A \cdot C_V \cdot A^T$$

Notice:

- the off-diagonal elements of covariance matrix C_V are the covariances of the $\mathbf{v}^{(i)}$ vectors ... $(V \cdot V^T)_{ab} = \sum_{i=1}^{k} v_a^{(i)} v_b^{(i)}$
- the off-diagonal elements of covariance matrix C_W are zero
- the eigenvectors of C_V form the rows of a new matrix A
- C_W is a diagonal matrix formed of the eigenvalues of C_V

PCA-based Transform

Algorithm:

1. collect the input data $\mathbf{b}^{(i)}$
2. form the matrix V
3. calculate the covariance matrix C_V
4. find eigenvectors and eigenvalues of C_V
5. use eigenvectors to form the transform matrix A
6. use A as a transform matrix to original data
7. get transformed decorrelated data: $\mathbf{w}^{(i)} = A(\mathbf{b}^{(i)} - \overline{\mathbf{b}})$

Notice: Red lines $\equiv$ Principal Component Analysis (PCA).

PCA-based Transform

An example

Let us submit the following 2D image to the PCA

2.5	2.4	0.5	0.7
2.2	2.9	1.9	2.2
3.1	3.0	2.3	2.7
2.0	1.6	1.0	1.1
1.5	1.6	1.1	0.9

Notice: Neighbouring pixels have usually similar value.

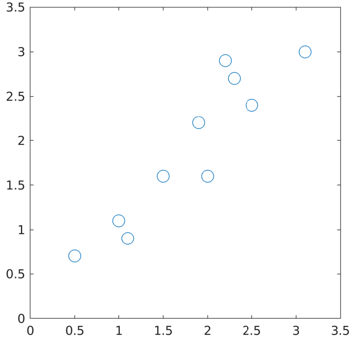
PCA-based Transform

An example

We have chosen $n = 2, k = 10$ and fetched k vectors $\mathbf{b}^{(i)}, i = 1, 2, \ldots, k$ in the following manner:

	X	Y
$\mathbf{b}^{(1)}$	2.5	2.4
$\mathbf{b}^{(2)}$	0.5	0.7
$\mathbf{b}^{(3)}$	2.2	2.9
$\mathbf{b}^{(4)}$	1.9	2.2
$\mathbf{b}^{(5)}$	3.1	3.0
$\mathbf{b}^{(6)}$	2.3	2.7
$\mathbf{b}^{(7)}$	2.0	1.6
$\mathbf{b}^{(8)}$	1.0	1.1
$\mathbf{b}^{(9)}$	1.5	1.6
$\mathbf{b}^{(10)}$	1.1	0.9

Correlation of the neighbouring intensities:



PCA-based Transform

An example

Modify the input datasets:

	X	Y		$X - \overline{X}$	$Y - \overline{Y}$
$\mathbf{b}^{(1)}$	2.5	2.4	$\mathbf{v}^{(1)}$	0.69	0.49
$\mathbf{b}^{(2)}$	0.5	0.7	$\mathbf{v}^{(2)}$	-1.31	-1.21
$\mathbf{b}^{(3)}$	2.2	2.9	$\mathbf{v}^{(3)}$	0.39	0.99
$\mathbf{b}^{(4)}$	1.9	2.2	$\mathbf{v}^{(4)}$	0.09	0.29
$\mathbf{b}^{(5)}$	3.1	3.0	$\mathbf{v}^{(5)}$	1.29	1.09
$\mathbf{b}^{(6)}$	2.3	2.7	$\mathbf{v}^{(6)}$	0.49	0.79
$\mathbf{b}^{(7)}$	2.0	1.6	$\mathbf{v}^{(7)}$	0.19	-0.31
$\mathbf{b}^{(8)}$	1.0	1.1	$\mathbf{v}^{(8)}$	-0.81	-0.81
$\mathbf{b}^{(9)}$	1.5	1.6	$\mathbf{v}^{(9)}$	-0.31	-0.31
$\mathbf{b}^{(10)}$	1.1	0.9	$\mathbf{v}^{(10)}$	-0.71	-1.01

PCA-based Transform

An example

Evaluate all the covariances $C_V(X,X)$, $C_V(X,Y)$, $C_V(Y,X)$, and $C_V(Y,Y)$ and form the covariance matrix C_V:

$$C_V = V \cdot V^T = \begin{bmatrix} 0.617 & 0.615 \\ 0.615 & 0.717 \end{bmatrix}$$

Since the covariance matrix is square, we can calculate the eigenvectors and eigenvalues for this matrix:

- eigenvalues

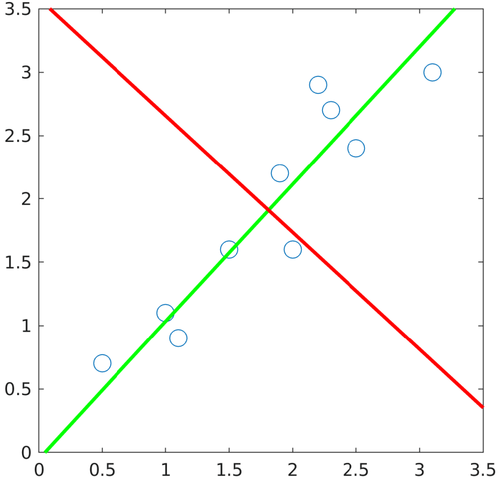
$$C_W = W \cdot W^T = \begin{pmatrix} 0.049 & 0 \\ 0 & 1.284 \end{pmatrix}$$

- eigenvectors

$$A = \begin{pmatrix} -0.735 & 0.678 \\ -0.678 & -0.735 \end{pmatrix}$$

PCA-based Transform

An example



PCA-based Transform

An example

The following statements are equal:

- the eigenvalue λ_j is the highest one
- the eigenvector e_j is more dominant
- the energy (the majority in information) is gathered in element $\mathbf{w}_j^{(i)}, i = 1, 2, \ldots, k$
- the element $\mathbf{w}_j^{(i)}, i = 1, 2, \ldots, k$ has the greatest variance

The following statements are also equal:

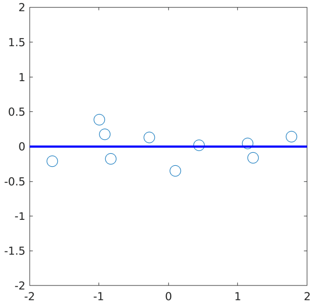
- the eigenvalue λ_l is the lowest one
- the eigenvector e_l is practically useless

PCA-based Transform

An example

Transformed (decorrelated) data

	X'	Y'
$\mathbf{w}^{(1)}$	-0.827	-0.175
$\mathbf{w}^{(2)}$	1.777	0.142
$\mathbf{w}^{(3)}$	-0.992	0.384
$\mathbf{w}^{(4)}$	-0.274	0.130
$\mathbf{w}^{(5)}$	-1.675	-0.209
$\mathbf{w}^{(6)}$	-0.912	0.175
$\mathbf{w}^{(7)}$	0.099	-0.349
$\mathbf{w}^{(8)}$	1.144	0.046
$\mathbf{w}^{(9)}$	0.438	0.017
$\mathbf{w}^{(10)}$	1.223	-0.162

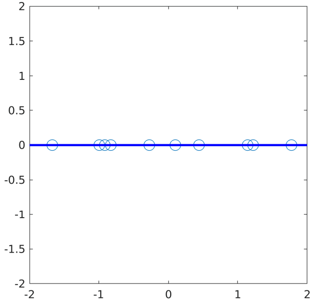


PCA-based Transform

An example

Clear less important component

	X'	Y'
$\mathbf{w}^{(1)}$	-0.827	0.000
$\mathbf{w}^{(2)}$	1.777	0.000
$\mathbf{w}^{(3)}$	-0.992	0.000
$\mathbf{w}^{(4)}$	-0.274	0.000
$\mathbf{w}^{(5)}$	-1.675	0.000
$\mathbf{w}^{(6)}$	-0.912	0.000
$\mathbf{w}^{(7)}$	0.099	0.000
$\mathbf{w}^{(8)}$	1.144	0.000
$\mathbf{w}^{(9)}$	0.438	0.000
$\mathbf{w}^{(10)}$	1.223	0.000



Transformed back with A^{-1}

	oldX	*oldY*	*newX*	*newY*
$\mathbf{b}^{(1)}$	2.5	2.4	2.42	2.47
$\mathbf{b}^{(2)}$	0.5	0.7	0.50	0.71
$\mathbf{b}^{(3)}$	2.2	2.9	2.54	2.58
$\mathbf{b}^{(4)}$	1.9	2.2	2.01	2.09
$\mathbf{b}^{(5)}$	3.1	3.0	3.04	3.05
$\mathbf{b}^{(6)}$	2.3	2.7	2.48	2.53
$\mathbf{b}^{(7)}$	2.0	1.6	1.74	1.84
$\mathbf{b}^{(8)}$	1.0	1.1	0.97	1.13
$\mathbf{b}^{(9)}$	1.5	1.6	1.49	1.61
$\mathbf{b}^{(10)}$	1.1	0.9	0.91	1.08

PCA-based Transform

Properties:

- optimal decorrelation method (see the example)
- too heavy for evaluation (searching for eigenvalues and eigenvectors)
- matrix A is data dependent (cannot be precomputed)
- all the eigenvectors must be kept for inverse transform

Conclusion: rather theoretical method

Discrete Cosine Transform (DCT)

DFT → DCT

Let $f = [f(0) \quad f(1) \ \ldots \ f(N-1)]$ be a discrete signal.

Let us modify it as follows:

$$
\begin{array}{rcl}
 & & f(0) \quad f(1) \quad f(2) \quad \ldots \quad f(N-1) \\
 & \Downarrow & \text{mirror} \\
f(N-1) \ \ldots \ f(1)\ f(0) & & f(0)\ f(1)\ f(2) \ \ldots \ f(N-1) \\
 & \Downarrow & \text{insert zeros} \\
f(N-1)\ 0 \ \ldots \ 0\ f(1)\ 0\ f(0) & 0 & f(0)\ 0\ f(1)\ 0\ f(2) \ \ldots \ 0 \ \ldots \ f(N-1)\ 0 \\
 & \Downarrow & \text{DFT domain is periodical} \\
 & 0 & f(0)\ 0\ f(1)\ 0\ f(2) \ \ldots \ 0 \ \ldots \ f(N-1)\ 0\ f(N-1)\ 0 \ \ldots \ 0\ f(1)\ 0\ f(0) \\
 & \Downarrow & \text{name substitution} \\
 & c(0) & c(1) \quad c(2) \quad c(3) \quad c(4) \ \ldots \ c(4N-1)
\end{array}
$$

The relationship between signal 'c' and 'f':

$$
\begin{aligned}
c(2n) &= 0 \quad \text{iff} \quad 0 \leq n < N \\
c(2n+1) &= f(n) \quad \text{iff} \quad 0 \leq n < N \\
c(4N-n) &= c(n) \quad \text{iff} \quad 0 < n < 2N
\end{aligned}
$$

The basic properties of signal 'c':

- c is even (ready for DFT working over real even data)
- $|c| = 4N$

Discrete Cosine Transform (DCT)

DFT $\rightarrow$ DCT

Let us apply DFT to c:

$$
\begin{aligned}
\mathcal{C}(k) &= \sum_{j=0}^{4N-1} c(j) e^{-\frac{2\pi ijk}{4N}} = \text{/symmetry/} = \sum_{j=0}^{2N-1} c(j) \left[e^{-\frac{2\pi ijk}{4N}} + e^{-\frac{2\pi i(4N-j)k}{4N}} \right] \\
&= \sum_{j=0}^{2N-1} c(j) \left[e^{-\frac{2\pi ijk}{4N}} + e^{\frac{2\pi ijk}{4N}} \right] \text{/Euler-Moivre eq./} \\
&= \sum_{j=0}^{2N-1} c(j) \left[\left(\cos \frac{2\pi jk}{4N} - i \sin \frac{2\pi jk}{4N} \right) + \left(\cos \frac{2\pi jk}{4N} + i \sin \frac{2\pi jk}{4N} \right) \right] \\
&= \sum_{j=0}^{2N-1} c(j) \left[2 \cos \frac{2\pi jk}{4N} \right] \\
&= \dots
\end{aligned}
$$

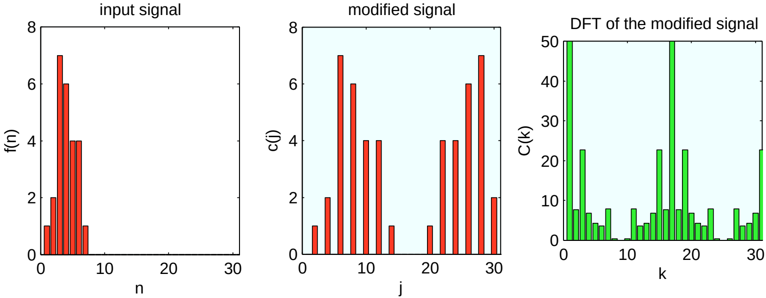
Discrete Cosine Transform (DCT)

DFT $\rightarrow$ DCT

$$
\begin{aligned}
\mathcal{C}(k) &= \sum_{j=0}^{2N-1} c(j)\left[2\cos\frac{2\pi jk}{4N}\right] && \text{/separating odd and even items/} \\
&= \sum_{l=0}^{N-1} 2c(2l)\left[\cos\frac{2\pi 2lk}{4N}\right] && \text{/j=2l, } l \in \mathbb{N}\text{/} \\
&\quad + \sum_{l=0}^{N-1} 2c(2l+1)\left[\cos\frac{2\pi(2l+1)k}{4N}\right] && \text{/j=2l+1, } l \in \mathbb{N}\text{/} \\
&= \sum_{l=0}^{N-1} 2f(l)\left[\cos\frac{(2l+1)\pi k}{2N}\right]
\end{aligned}
$$

Signal $\mathcal{C}(k)$ is:

- even, because 'c' is even
- twice replicated, because 'c' is stretched by zeros



Notice: Only the coefficients $\mathcal{C}(k), k = \{0, 1, 2, \ldots, N-1\}$ are used.

This version of DCT is also known as DCT-II.

Discrete Cosine Transform (DCT)

Definition

Forward 1D-DCT:

$$\mathcal{C}(k) = \alpha(k) \sum_{l=0}^{N-1} f(l) \left[\cos \frac{(2l+1)\pi k}{2N} \right], \quad k = \{0, 1, 2, \ldots, N-1\}$$

where

$$\alpha(k) = \begin{cases} \sqrt{\frac{1}{N}} & \text{iff} \quad k = 0 \\ \sqrt{\frac{2}{N}} & \text{iff} \quad k \neq 0 \end{cases}$$

Inverse 1D-DCT:

$$f(l) = \sum_{k=0}^{N-1} \alpha(k)\mathcal{C}(k) \left[\cos \frac{(2l+1)\pi k}{2N} \right], \quad l = \{0, 1, 2, \ldots, N-1\}$$

Basic properties:

- basis formed of sampled cosine waves only
- no complex numbers

Fast Discrete Cosine Transform (F-DCT)

Derivation

Let us recombine the input signal:

$$\begin{aligned} y(l) &= f(2l) \\ y(N-1-l) &= f(2l+1) \quad (l = 0, \ldots, N/2-1) \end{aligned}$$

Example: `f = [1 2 3 4 5 6 7 8]` $\rightarrow$ `y = [1 3 5 7 8 6 4 2]`

Applying DCT on signal f we can deduce:

$$\begin{aligned} \mathcal{C}(k) &= \alpha(k) \sum_{l=0}^{N-1} f(l) \cos\left(\frac{(2l+1)\pi k}{2N}\right) \quad /f \rightarrow y/ = \cdots = \\ &= \alpha(k) \sum_{l=0}^{N-1} y(l) \cos\left(\frac{(4l+1)\pi k}{2N}\right) \end{aligned}$$

Fast Discrete Cosine Transform (F-DCT)

Derivation (cont'd)

Let us apply DFT on signal y:

$$
\begin{aligned}
\mathcal{Y}(k) &= \sum_{l=0}^{N-1} y(l) e^{\frac{-2\pi i k l}{N}} \\
&= \sum_{l=0}^{N-1} y(l) \left[\cos\frac{2\pi k l}{N} - i \sin\frac{2\pi k l}{N}\right] \;/\cdot e^{-\frac{\pi i k}{2N}}\;/ \\
\text{Real}\left[e^{-\frac{\pi i k}{2N}} \mathcal{Y}(k)\right] &= \sum_{l=0}^{N-1} y(l) \left[\cos\frac{2\pi k l}{N} \cos\frac{k\pi}{2N} - \sin\frac{2\pi k l}{N} \sin\frac{k\pi}{2N}\right] \\
&= \sum_{l=0}^{N-1} y(l) \cos\frac{(4l+1)k\pi}{2N} = \mathcal{C}(k)/\alpha(k) \\
\text{Imag}\left[e^{-\frac{\pi i k}{2N}} \mathcal{Y}(k)\right] &= \text{omitted}
\end{aligned}
$$

$$
\mathcal{C}(k) = \alpha(k) \text{Real}\left[e^{-\frac{\pi i k}{2N}} \mathcal{Y}(k)\right]
$$

Fast Discrete Cosine Transform (F-DCT)

Algorithm

1. recombine input sequence f of length N to get y:

$$\begin{aligned} y(l) &= f(2l) \\ y(N-1-l) &= f(2l+1) \quad (l = 0, \ldots, N/2 - 1) \end{aligned}$$

2. apply FFT to y:

$$\mathcal{Y} = \mathrm{FFT}(y)$$

3. for each $k = 0, \ldots, N-1$ do:
   1. multiply the k-th Fourier coefficient by factor $e^{-\frac{\pi i k}{2N}}$:

$$\mathcal{Y}'(k) = e^{-\frac{\pi i k}{2N}} \mathcal{Y}(k)$$

   2. fetch only real part from each Fourier coefficient and normalize the results:

$$\mathcal{C}(k) = \alpha(k) \mathrm{Real}\left[\mathcal{Y}'(k)\right]$$

Forward 2D-DCT:

$$\mathcal{C}(u,v) = \alpha(u)\alpha(v)\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} f(k,l)\left[\cos\frac{(2k+1)\pi u}{2N}\cos\frac{(2l+1)\pi v}{2N}\right]$$

$$\text{where} \quad u,v = \{0,1,2,\dots,N-1\}$$

Inverse 2D-DCT:

$$f(k,l) = \sum_{u=0}^{N-1}\sum_{v=0}^{N-1} \alpha(u)\alpha(v)\mathcal{C}(u,v)\left[\cos\frac{(2k+1)\pi u}{2N}\cos\frac{(2l+1)\pi v}{2N}\right]$$

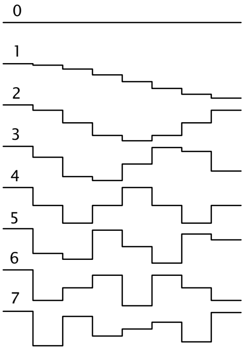
Basic properties:

- 2D-DCT it is simply an extension of 1D-DCT
- separable

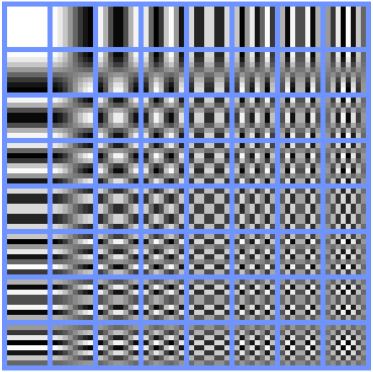
2D Discrete Cosine Transform (2D-DCT)

Basis functions

1D-DCT
8 basis 1D functions
($N = 8$)



2D-DCT
64 basis 2D functions
($N \times N = 8 \times 8$)



Discrete Cosine Transform (DCT)

Properties

- Almost as efficient as PCA in term of decorrelation optimality.
- The transform matrix can be easily prepared without having the data.
- Unlike DFT, DCT works with real data.
- As its is derived directly from FFT, there exists fast alternative for DCT.
- Regarding its construction, it works with symmetric data.

Walsh-Hadamard Transform

Jacques Salomon Hadamard (1865 – 1963)

Walsh-Hadamard Transform

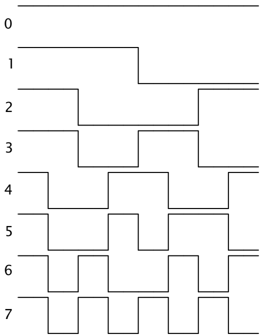
Similarly to DFT we can define Hadamard matrix which defines the transform:

$$
\begin{aligned}
H_m &= \frac{1}{\sqrt{2}} \begin{pmatrix} H_{m-1} & H_{m-1} \\ H_{m-1} & -H_{m-1} \end{pmatrix} \\
H_1 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \\
H_0 &= +1
\end{aligned}
$$

An example:

$$
H_2 = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}
$$

An 8-samples long Walsh functions:



Walsh-Hadamard Transform

Formal definition:

$$\mathcal{H}(k) = \frac{1}{N} \sum_{n=0}^{N-1} f(n) \prod_{i=0}^{m-1} (-1)^{B(m,i,n,k)}$$

with $k \in \{0, 1, \ldots, N-1\}$, $N = 2^m$, and $B(m, i, n, k) = b_i(n) b_{m-1-i}(k)$, where $b_k(v)$ denotes the k-th bit in the binary representation of a non-negative integer v.

Notice: Similarly to FT we can define inverse (IWHT) or fast (FWHT) Walsh-Hadamard transform.

Walsh-Hadamard Transform

Properties:

- Unlike to DCT the basis functions contain plus and minus ones only.
- So-called *Walsh functions* are used as a basis functions.
- Any two basis functions are orthogonal.
- In real space only (like DCT).
- Worse approximation of PCA than DCT.
- Wide application in digital communications.
- One can implement the WHT on smaller, cheaper hardware.

Notice: Magnitude in WHT is affected by phase shifts in the signal! Typically, orthogonality is broken.

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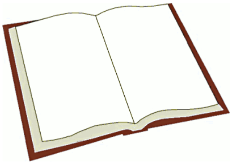

You should know the answers ...

- Explain the difference between correlated and decorrelated signal.
- How does the PCA decorrelate the given signal?
- How do we get the PCA transformation matrix A in practice?
- What is the content of matrix A used in PCA transform?
- Provide your own example (different from the examples presented in the lecture) suitable for PCA transform.
- Explain the relationship between DFT and DCT.
- Describe the F-DCT algorithm and compute F-DCT(`[1 6 6 1]`).
- Analyze the ability to compact the energy for the following transforms: PCA, DFT, DCT, WHT