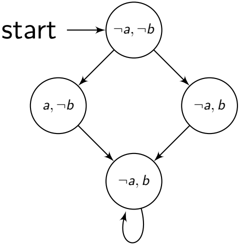
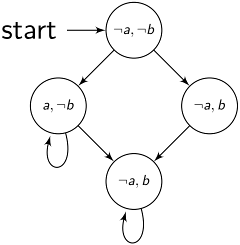
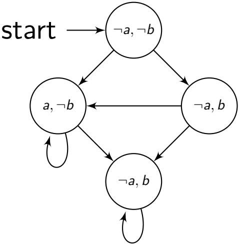
LTL examples

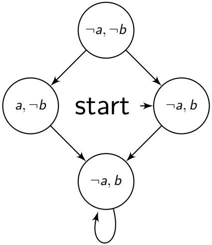
What formulas are true on what systems?
If not, show a counterexample.

- $\neg a \wedge b$
- $F\ a$
- $\neg a\ U\ b$
- $G\ b$
- $F\ G\ b$









LTL examples

- $X\ rain$
- $F\ rain$
- $pick\text{-}up\ R\ kin\text{-}gar$
- $G(drop\text{-}off \implies (kin\text{-}gar\ U\ pick\text{-}up))$
- $G\,(\neg(cs_1 \wedge cs_2))$
- $G\,(req \implies F\ resp)$ ... Does it guarantee that $\#_{req} = \#_{resp}$?
- $G\ F\ chocolate$
- $(G\ F\ req) \implies (G\ F\ resp)$
- $sin \implies (F\ G\ hell)$
- $F(sin \wedge (\neg confession\ U\ death)) \implies (F\ G\ hell)$

LTL properties - example

You have two fishes, say Alice (A) and Bob (B). There is an aquarium divided into two parts: left (L) and right (R). Both fish start on the right side of the aquarium and do the following sequence of steps (independently): They move to the left, eat, move back to the right. Formulate using LTL:

- Whenever Alice eats, she is on the left.
- Whenever Bob is on the left, he will eat eventually.
- Whenever Bob eats, he will immediately go to the left.
- If Alice does not eat before Bob, she will never eat.
- Alice and Bob will never be on the same side from some point.
- Bob chases Alice until they both eat together.

LTL properties - negation

Note and discuss that

- $\psi \implies \varphi\, U\, \psi$
- $\psi\, U\, \psi \equiv \psi$
- $true\, U\, \varphi \equiv F\varphi$
- $(\neg\psi)\, U\, \psi \equiv F\psi$
- $\neg(X\, \varphi) \equiv X\, \neg\varphi$
- $\neg(G\, \varphi) \equiv F\, \neg\varphi$
- $\varphi\, W\, \psi \equiv \varphi\, U\, \psi \;\vee\; G\, \varphi$
- $\neg(\varphi\, R\, \psi) \equiv \neg\varphi\, U\, \neg\psi$
- $\varphi\, R\, \psi \equiv \psi\, W\, (\varphi \wedge \psi)$

LTL - temporal properties

Note and discuss that

- $F\,\varphi \equiv F\,(F\,\varphi)$
- $G\,\varphi \equiv G\,(G\,\varphi)$
- $\varphi\,U\,\psi \equiv \varphi\,U\,(\varphi\,U\,\psi)$
- $\varphi\,U\,\psi \equiv \psi \vee (\varphi \wedge X\,(\varphi\,U\,\psi))$
- $\varphi\,W\,\psi \equiv \psi \vee (\varphi \wedge X\,(\varphi\,W\,\psi))$
- $\varphi\,R\,\psi \equiv \psi \wedge (\varphi \vee X\,(\varphi\,R\,\psi))$

- $G\,\varphi \equiv \varphi \wedge X\,(G\,\varphi)$
- $F\,\varphi \equiv \varphi \vee X\,(F\,\varphi)$
- $X\,F\,\varphi \equiv F\,(X\,\varphi)$
- $X\,G\,\varphi \equiv G\,(X\,\varphi)$
- $X(\varphi\,U\,\psi) \equiv (X\varphi)\,U\,(X\psi)$

LTL properties - distributivity questions

Is it true that ...

- $X(\varphi \vee \psi) \stackrel{?}{\equiv} X\varphi \vee X\psi$
- $X(\varphi \wedge \psi) \stackrel{?}{\equiv} X\varphi \wedge X\psi$
- $F(\varphi \vee \psi) \stackrel{?}{\equiv} F\varphi \vee F\psi$ $\quad GF(\varphi \vee \psi) \stackrel{?}{\equiv} GF\varphi \vee GF\psi$
- $F(\varphi \wedge \psi) \stackrel{?}{\equiv} F\varphi \wedge F\psi$ $\quad GF(\varphi \wedge \psi) \stackrel{?}{\equiv} GF\varphi \wedge GF\psi$
- $G(\varphi \vee \psi) \stackrel{?}{\equiv} G\varphi \vee G\psi$ $\quad FG(\varphi \vee \psi) \stackrel{?}{\equiv} FG\varphi \vee FG\psi$
- $G(\varphi \wedge \psi) \stackrel{?}{\equiv} G\varphi \wedge G\psi$ $\quad FG(\varphi \wedge \psi) \stackrel{?}{\equiv} FG\varphi \wedge FG\psi$
- $\varphi\, U\, (\psi_1 \vee \psi_2) \stackrel{?}{\equiv} (\varphi\, U\, \psi_1) \vee (\varphi\, U\, \psi_2)$
- $\varphi\, U\, (\psi_1 \wedge \psi_2) \stackrel{?}{\equiv} (\varphi\, U\, \psi_1) \wedge (\varphi\, U\, \psi_2)$
- $(\varphi_1 \vee \varphi_2)\, U\, \psi \stackrel{?}{\equiv} (\varphi_1\, U\, \psi) \vee (\varphi_2\, U\, \psi)$
- $(\varphi_1 \wedge \varphi_2)\, U\, \psi \stackrel{?}{\equiv} (\varphi_1\, U\, \psi) \wedge (\varphi_2\, U\, \psi)$