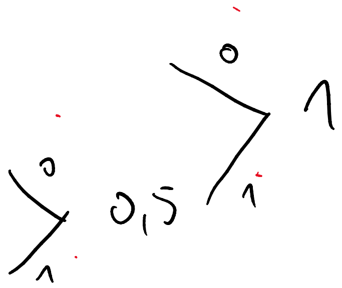
Consultation I

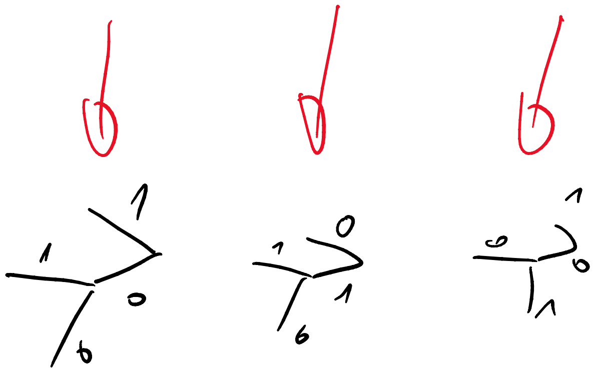
15 October 2020 09:24

a 0,5

b 0,25

c 0,25





b

A 0

B 10

C 11

1   0   1

0 1   1 1   0 0

0 0   1 0   0 1

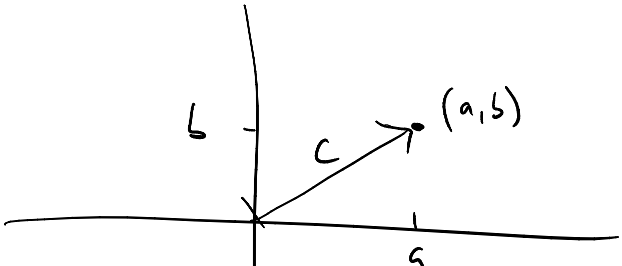
A B C A C A C B B

0 | 10 | 11 | 0 | 1 1 | 0 | 11 | 1 0 | 1 0

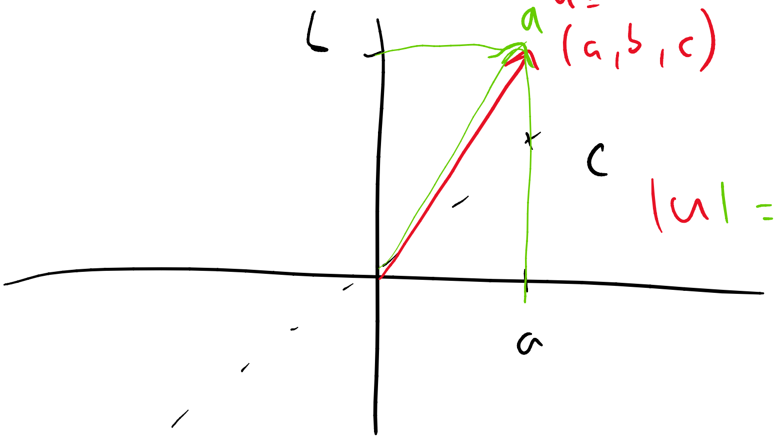
Lemma. *Let* $u_0, u_1, \ldots, u_m \in \mathbb{R}^n$ *such that* $\boxed{|u_i| = 1}$ *and* $\boxed{u_i \cdot u_j \leq 0}$ *for all* $0 \leq i < j \leq m$. *Then* $\boxed{m \leq 2n}$.

$u_i \in \mathbb{R}_n$ $\qquad u_i = (r_0, r_1, \ldots, r_{n-1})$

$\underbrace{u_0, \ldots, u_{m-1}}_{m \text{ vectors}}$ $\qquad |u_i| = 1 \quad |u_i| = \sqrt{r_0^2 + r_1^2 + \cdots + r_{n-1}^2}$



$$\sqrt{a^2+b^2} = c$$



$$|d| = \sqrt{a^2+b^2}$$

$$|u| = \sqrt{d^2+c^2} = \sqrt{a^2+b^2+c^2}$$

$$\forall_{i,j} \quad u_i \cdot u_j = \quad r_{i0}\cdot r_{j0} + r_{i1}\cdot r_{j1} + \dots + r_{i,n-1}\cdot r_{j,n-1}$$

$$\leq 0$$

$$C_n = \{0123,\ 1111,\ 2310\} \cong \{0000, \qquad \}$$

position 1<->4 ||}

$$\{3012,\ 1111,\ 0231\}$$

$$\pi_1 = (03) \quad ||\}$$

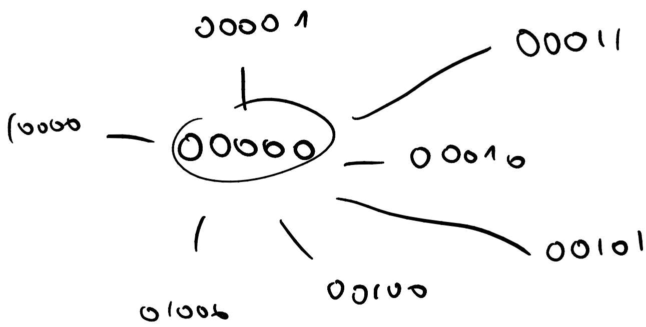
$$\{0012,\ 1111,\ 3231\}$$

$$\pi_3 = (1\,0)\ \|\}$$

$$\{0002,\ 1101,\ 3231\}$$

$$\pi_4 = (0,2)\ \|\}$$

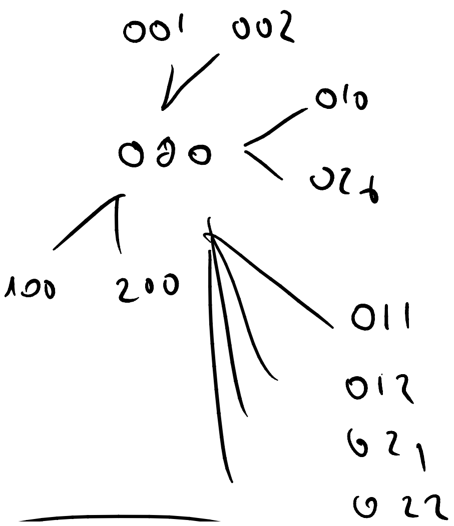
$$C_2 = \{0000,\ 1101,\ 3231\}$$



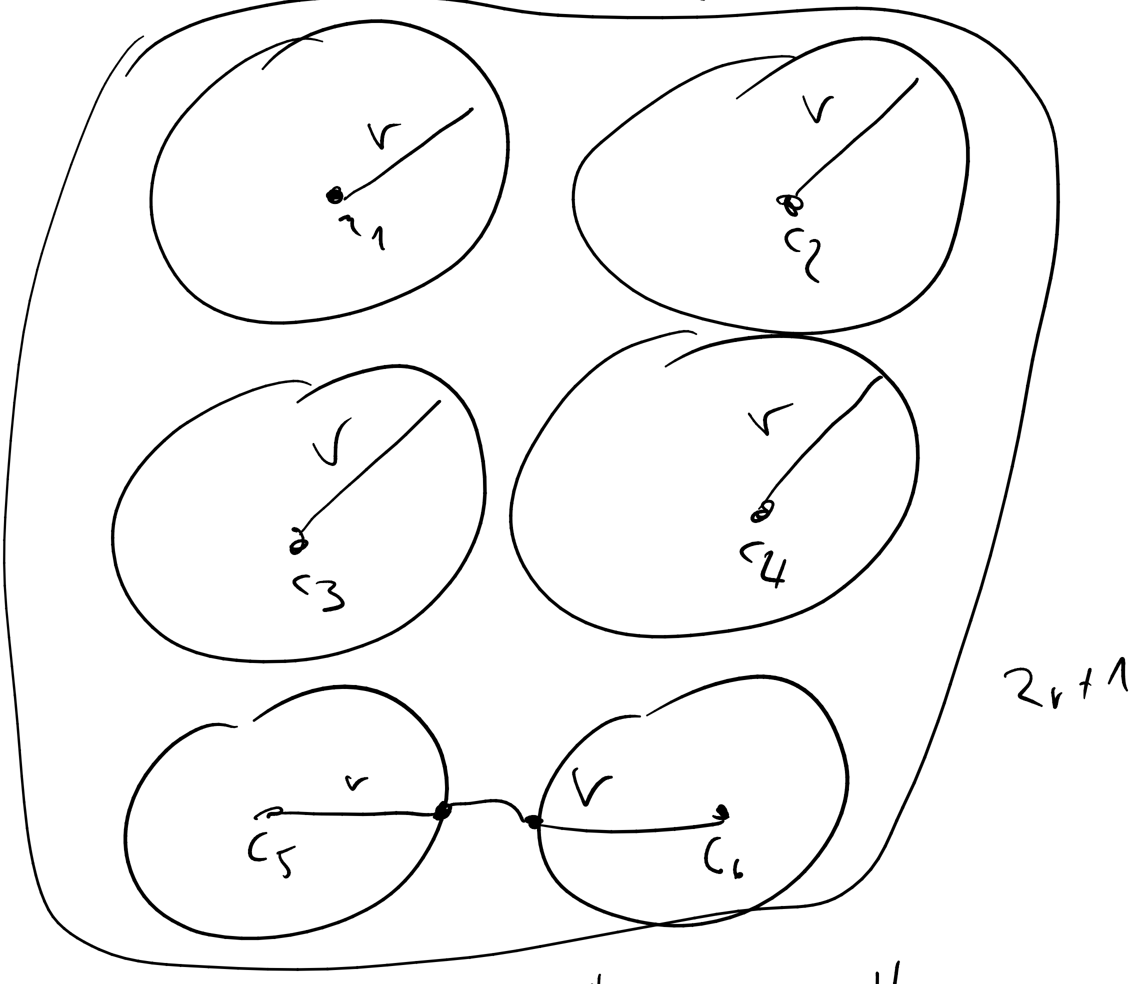
$\binom{5}{1}$ $\binom{5}{2}$

$$1+\binom{5}{1}+\binom{5}{2}$$

q-ary



$$1+\binom{3}{1}(q-1)+\binom{3}{2}(q-1)^2$$



Perfect code $R(c_i, r)$ – all codewords within distance r from c_i

$$= \sum_{i \in C} R(c_i, r) = \{0, 1, \dots, q-1\}^n$$

$C = (n, M, 2r+1)$ code

$\boxed{\text{Sphere packing bound}}$

$$M\left(1+\binom{n}{1}(q-1)+\binom{n}{2}(q-1)^2+\cdots+\binom{n}{r}(q-1)^r\right)$$

$$\leq 2^n$$

00000   00111   11011   11100 $(5,4,3)$

00001 $\boxed{01110}$

	00000	00111	11011	11100
01110	3	2	3	2

6

1 error can be corrected

$00000 \xrightarrow{1} \boxed{00001}$		$(1-p)^4 p$
$00111 \xrightarrow{2} 00001$		$(1-p)^3 p^2$
$11011 \xrightarrow{3} 00001$		$(1-p)^2 p^3$
$11100 \xrightarrow{4} 00001$		$(1-p) p^4$

$p < 1-p$

PML for $\boxed{\text{binary symmetric channel}}$

$\|$

Nearest neighbor decoding strategy