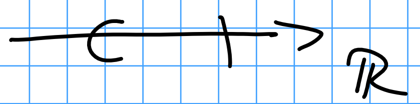
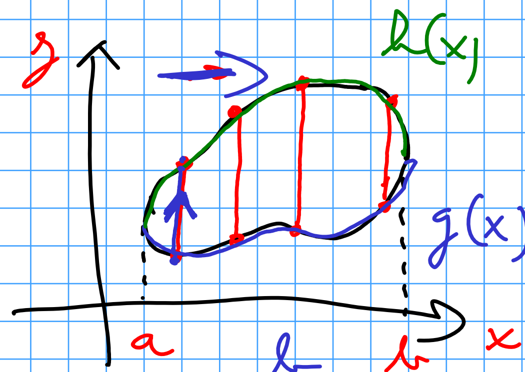
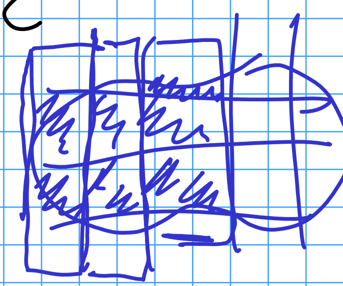
Integrální počet ve více prom.

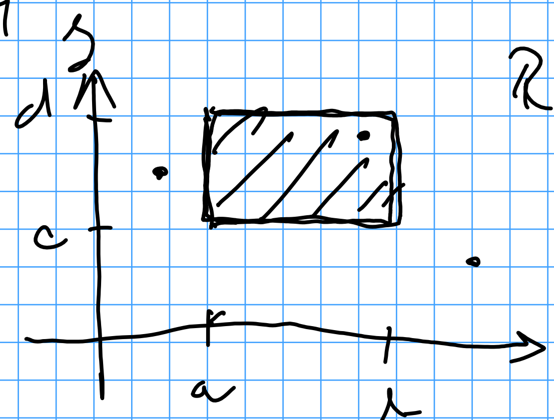




$\mathbb{R}^2$

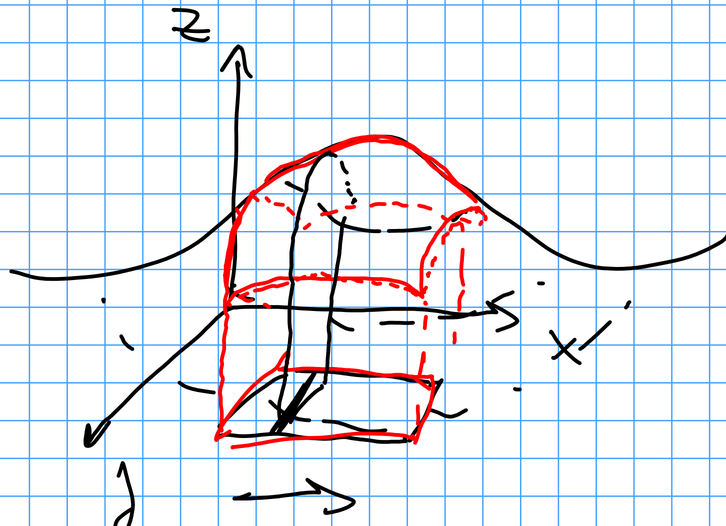


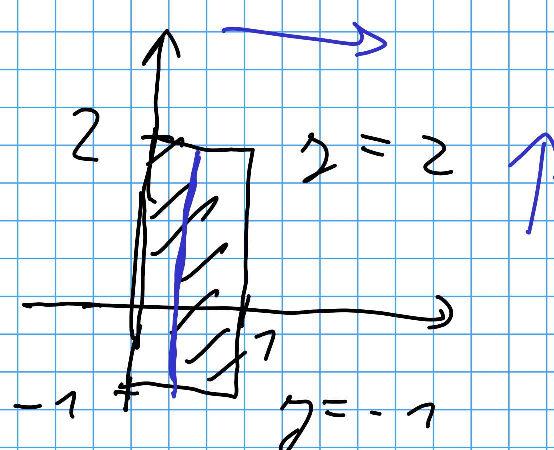
$$\iint_M f(x,y)\,dx\,dy = \int_a^b \left(\int_{g(x)}^{h(x)} f(x,y)\,dy \right) dx$$



$\mathbb{R}^2$

$[a,b] \times [c,d]$

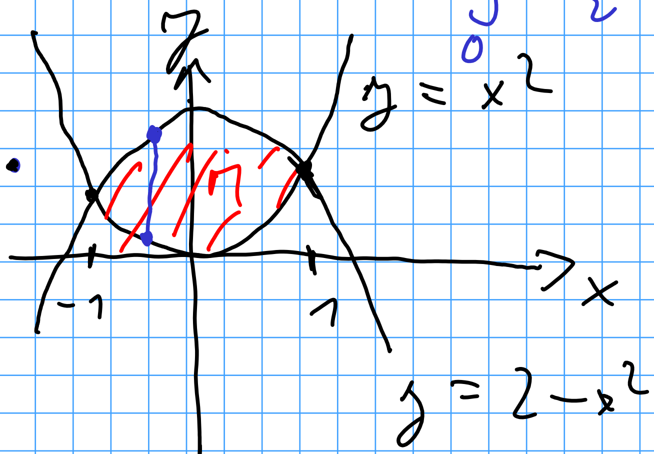




$$\iint_{[0,1]\times[-1,2]} x y \,dx\,dy = \int_0^1 \left(\int_{-1}^{2} x y \,dy \right) dx$$

$$= \int_0^1 \left[x \frac{y^2}{2} \right]_{-1}^{2} dx = \int_0^1 2x - \frac{1}{2} x \, dx$$

$$= \int_0^1 \frac{3}{2} x \, dx = \frac{3}{2} \left[\frac{x^2}{2} \right]_0^1 = \underline{\underline{\frac{3}{4}}} \left(- \frac{3}{2} \cdot 0 \right)$$



$$\iint_M x \, dx dy$$

$$x^2 = 2 - x^2$$

$$0 = 2 - 2x^2 = 2(1 - x^2)$$

$$= 2(1-x)(1+x)$$

$M: \; y \in [x^2, 2 - x^2]$

$\quad\; x \in [-1, 1]$

$$= \int_{-1}^{1} \left(\int_{x^2}^{2-x^2} x \, dy \right) dx$$

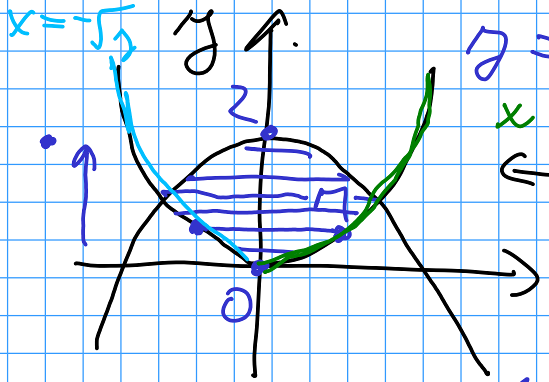
$$= \int_{-1}^{1} \left[x y \right]_{x^2}^{2-x^2} dx$$

$$= \int_{-1}^{1} x(2 - x^2) - x^3 \, dx$$

$$= \int_{-1}^{1} 2x - 2x^3 \, dx$$

$$= \left[x^2 - \frac{2}{4} x^4 \right]_{-1}^{1}$$

$$= 1 - \frac{1}{2} - \left(1 - \frac{1}{2} \right) = 0$$



$y = x^2$

$x = \sqrt{y}$ $\Leftarrow$

$y \in [0, 1]$ $\quad$ $y \in [1, 2]$

$x \in [-\sqrt{y}, \sqrt{y}]$ $\quad$ $x \in [-\sqrt{2-y}, \sqrt{2-y}]$

$y = 2 - x^2$

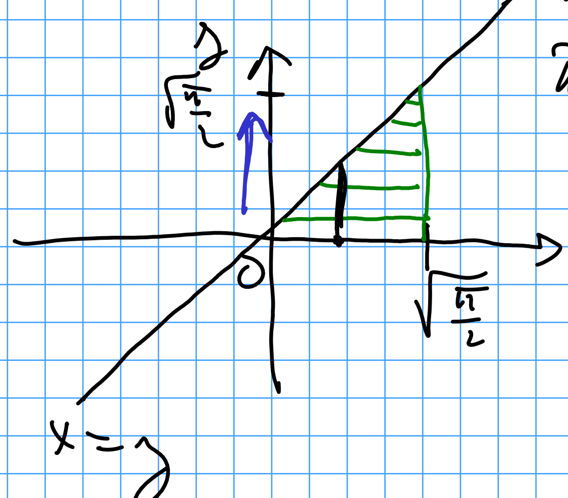
$$\iint_M f(x,y)\,dx = \int_0^1 \left(\int_{-\sqrt{y}}^{\sqrt{y}} x \, dx \right) dy + \int_1^2 \left(\int_{-\sqrt{2-y}}^{\sqrt{2-y}} x \, dx \right) dy$$

$$\left(\iint_{M_1 \cup M_2} f(x,y)\,dx\,dy = \iint_{M_1} f\,dx\,dy + \iint_{M_2} f\,dx\,dy \right)$$

$$\int_0^{\sqrt{\frac{\pi}{2}}} \left(\int_y^{\sqrt{\frac{\pi}{2}}} y^2 \sin(x^2)\,dx \right) dy = \iint_M y^2 \sin x^2 \, dx\,dy$$

$= (*)$

$\mathbb{R}^2$



M: $y \in \left[0, \sqrt{\frac{\pi}{2}}\right]$

$x \in \left[y, \sqrt{\frac{\pi}{2}}\right]$

$x = y$

$x \in \left[0, \sqrt{\frac{\pi}{2}}\right]$

$y \in [0, x]$

$$(*) = \int_0^{\sqrt{\pi/2}} \left(\int_0^x y^2 \sin x^2 \, dy \right) dx$$

$$= \int_0^{\sqrt{\pi/2}} \left[\frac{y^3}{3} \sin x^2 \right]_0^x dx$$

$$\frac{1}{6} \cdot 2x \cdot x^2$$

$$= \int_0^{\sqrt{\pi/2}} \frac{x^3}{3} \sin x^2 \, dx \quad \left| \begin{array}{l} t = x^2 \\ dt = 2x \, dx \\ 0 \to 0 \\ \sqrt{\frac{\pi}{2}} \to \frac{\pi}{2} \end{array} \right|$$

$$= \frac{1}{6} \int_0^{\pi/2} t \sin t \, dt$$

$$\overset{p-p}{=} \ldots\ldots$$